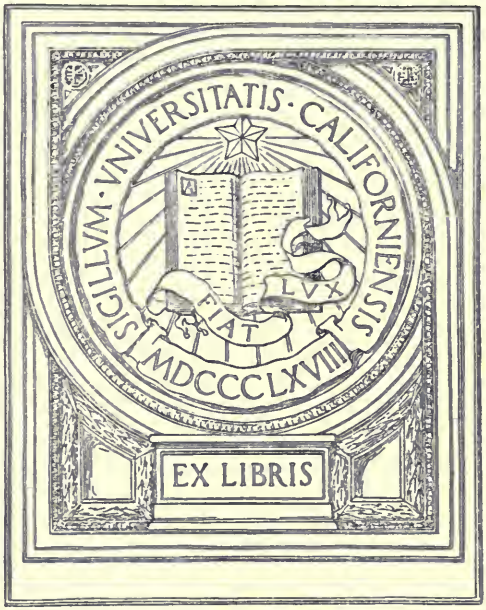
UC SOUTHERN REGIONAL LIBRARY FACILITY
A 000 768 739 5

UNIVERSITY OF CALIFORNIA
LOS ANGELES



GIFT OF
Col. Glen F. Jenks

ORDNANCE DEPARTMENT DOCUMENT NO. 2035-A

THEORY AND DESIGN OF RECOIL SYSTEMS AND GUN CARRIAGES

(CHAPTER — VIII)

Prepared in the
Office of the Chief of Ordnance

JANUARY 1922

CONFIDENTIAL

ENGINEER REPRODUCTION PLANT
WASHINGTON BARRACKS, D.C.

THEORY AND DESIGN
OF
RECOIL SYSTEMS
AND
GUN CARRIAGES

(Chapters I — XIII)

Prepared in the
Office of the Chief of Ordnance

Engineering
Library
UF
643
U58t

ORDNANCE DEPARTMENT

Document No.2035-A

Office of the Chief of Ordnance

Digitized by the Internet Archive
in 2007 with funding from
Microsoft Corporation

http://www.archive.org/details/theorydesignofre00unitiala

WAR DEPARTMENT,
Washington, February 1922.

The following publication, entitled "Theory and Design of Recoil Systems and Gun Carriages", is published for the information and guidance of all students of the Ordnance training schools, military academies, and other similar educational organizations.

The contents are strictly of a Confidential nature and should not be republished without authority of this Department.

C. C. WILLIAMS,
Major General, Chief of Ordnance, U.S.A.

523414

HYDRO PNEUMATIC RECOIL SYSTEMS.

INTRODUCTION. The fundamental advantages of these recoil systems are smoothness of operation with consequent durability and ability to meet with requirements of service within space and weight limitations. Light and small recuperator forgings are made possible by the use of high pressures, both in the recuperator cylinder as well as in the hydraulic. To obtain and hold these pressures, special systems of packing are necessary and successful manufacture depends largely upon use of proper packing.

RECOIL SYSTEM WITH CONSTANT ORIFICE FIXED LENGTH OF RECOIL. This system consists usually of two cylinders; the hydraulic brake cylinder and a recuperator cylinder containing a floating piston which separates the oil and air, this cylinder communicating with the hydraulic cylinder by means of a large opening at one end of the cylinder.

The floating piston has a long buffer rod which passes through a fixed diaphragm in the recuperator cylinder and enters a buffer at its end, serving a double purpose since it gives constant throttling through the diaphragm on recoil and serves as a throttling buffer on counter recoil.

During recoil the constant orifice about the buffer rod at the diaphragm of the recuperator cylinder produces a throttling of the oil and a consequent drop of pressure due to the constant orifice.

During counter recoil the oil passes through a constant orifice produced by constant throttling grooves in the buffer chamber. Toward the end of recoil the depth of these grooves vary, causing a varying orifice. The throttling is so designed as to cause a sufficient drop of pressure from the air, to produce a pressure against the hydraulic piston just sufficient to balance the total friction and thus nearly maintain uniform motion through a considerable part of counter recoil.

At the end of c'recoil where we have the decreasing variable orifice, the pressure is rapidly reduced to zero; thus the recoiling mass is brought to rest practically by only the total friction of the guides, stuffing box and piston frictions.

A characteristic of a constant orifice in a recoil system is to give a peak in the pressure curve in the region of the maximum velocity of recoil. The maximum velocity and the consequent maximum drop of pressure due to throttling occurs, fortunately at the beginning of recoil where the air pressure is at its minimum. As the gun recoils, the drop of pressure decreases due to the decreased velocity and consequent less throttling through the orifice, whereas the air pressure rises, and since at all times the hydraulic pressure is the sum of the air and drop of pressure due to throttling, we have a tendency to maintain a constant hydraulic pressure in the brake cylinder. This effect is especially true with a fairly long recoil. If, however, the recoil is shortened, the peak effect due to the throttling at the beginning of recoil is much greater than the increase of air pressure at the end of recoil.

The type of system now under discussion has a single length of recoil for all elevations which is made sufficiently long for stability at zero degrees, while the ratio of final air pressure to initial

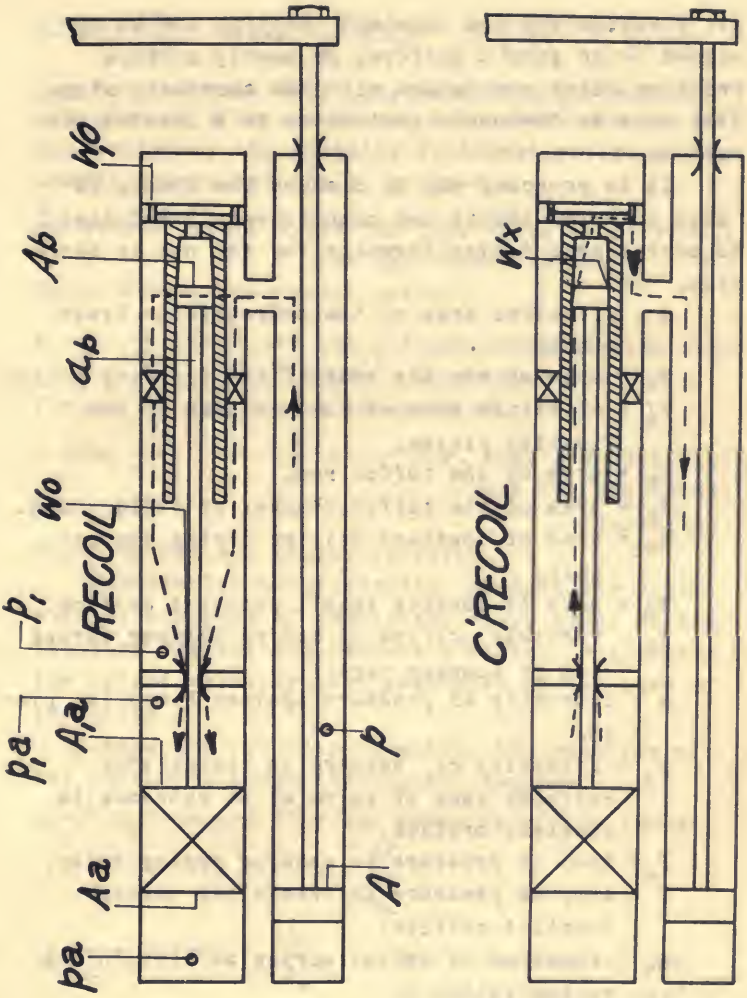


Fig.1

air pressure and the constant orifice are so designed as to give a uniform, or nearly uniform pressure curve consistent with the stability slope. This type is obviously restricted to a limited elevation.

It is proposed now to discuss the theory design both for recoil and counter recoil and then to approximate design formulae for the use in design, Let

A = effective area of the hydraulic or brake piston

A_a = area of the air side of the floating piston.

A_a' = effective area of the oil side of the floating piston.

a_b = area of the buffer rod.

A_b = area of the buffer chamber or buffer head.

W_o = area of constant orifice during the recoil,

W_o' = area of counter recoil constant orifice.

W_x = variable orifice in buffer chamber during end of counter recoil.

p = intensity of pressure against hydraulic piston

p_1 = intensity of ressure in recuperator cylinder back of valve or at entrance to constant orifice.

P_1 = drop of pressure in passing spring valve.

P = drop of pressure in throttling through constant orifice.

R_v = reaction of spiral spring at lift "h" on spring valve.

d = spring constant of the spiral springs

h_o = initial compression of spring

a_s = area of base of spring valve

c = circumference of spring valve = πd

If K equals the total resistance to recoil, we have during the powder pressure period, the total powder force being F_1

$F-K=M_r \frac{d^2x}{dt^2}$ hence $\frac{F-K}{M_r}\Delta t=\Delta v$ or $\frac{F}{M_r}\Delta t-\frac{K}{M_r}\Delta t=\Delta v$

but for the same time intervals when the recoiling mass if free, assuming the powder force to be unaffected by the slightly different motion of the powder charge and shell, we have

$\frac{F}{M_r}\Delta t=\Delta v_f$ where v_f is the free velocity of recoil for the recoiling mass.

Hence during the powder period,

$\Delta v_f - \frac{K}{M_r}\Delta t=\Delta v$ and the corresponding displacement of recoil, becomes, $\Delta x = v \Delta t$.

During the subsequent retardation,

$-K=M_r \frac{d^2x}{dt^2}$ or $-\frac{K}{M_r}\Delta t = \Delta v$, when $K = pA+R-W_r\sin\phi$ and R = the total guide and stuffing box and piston friction.

Considering now figure(1) we have the pressure at the base of the spring valve equal to the pressure against the hydraulic piston, neglecting the slight velocity head generated at the base of the valve.

From the law of continuity, the quantity of oil passing through the valves, becomes,

$Q_v=AV-A_bV'$ where V' = the velocity of the floating piston = $\frac{AV}{A_a}$

thus $Q_v = AV(1 - \frac{A_b}{A_a})$ Now if we neglect the dynamic reaction against the valve which is small, we have $P_1 a_s=R_v=S(h+h_o)$ but P_1 for the four valves, becomes,

$P = \frac{K^2Q_v^2}{2800\times(ch)^2} = \frac{S(h+h_o)}{a_s}$ hence the valve lift is obtained from the cubic equation:

$h^2(h+h) = \frac{K^2Q_v^2}{2800\times S\times O^2}$ and the corresponding drop of pressure by substituting

the K obtained from the above equation in the formula

$$p-p_1=P_1 = \frac{S}{a_s}(h+h_o)$$

In general this refinement may be omitted, since the drop through the valves is at a maximum, but 10% of the total drop, and the movement of the spring is at a maximum not greater than one third its total deflection to solid height, thus making a gross error of less than 3% if we consider the spring reaction that at assembled height. Consequently in the calculation of the pressure and velocity curves we will assume $P_1 a = S\ h_o = p-p_1 =$ a constant.

Now the quantity of oil passing through the constant orifice, is equal to that passing through the valves plus the quantity of oil displaced by the annular area about the buffer rod in the buffer chamber, that is $Q_o=(AV-A_bV')+(A_b-a_b)V' = AV-a_bV'$

$$= AV(1-\frac{a_b}{A_a})\simeq(AV)\frac{A'_a}{A_a}$$

A check on this valve may be obtained by noting that of the total quantity of oil displaced by the floating piston $A_aV'=AV$ but $(A_a-a_b)V'=A'_aV'$ passes through the constant orifice, and we note $V' = \frac{AV}{A_a}$

hence as above, $Q_o = (\frac{A_a-a_b}{A_a})AV$

The drop of pressure due to throttling through the constant orifice, becomes,

$$P_o = \frac{K_o^2 Q_o^2}{175W_o^2} = p_1-p'_a \quad \text{Therefore, we have, } p=P_1+P_o+p'$$

$$= Sh_o + \frac{K_o^2Q_o^2}{175W_a^2} + p'_a$$

To obtain p'_a we must further consider the equilibrium of the floating piston, neglecting the effect of its inertia during the acceleration and retardation as

being small. The forces in the direction of motion acting on the floating piston, are:

(1) The air pressure on the air side of the piston resisting the motion.

(2) The packing friction resisting motion.

(3) The oil pressure on the oil side of the floating piston,

(4) A resisting pressure on the annular area of the buffer.

(5) The pressure on the base of the buffer.

The pressure in the buffer chamber is assumed the same as the pressure in the hydraulic or recoil cylinder since by computation it is found that the drop in pressure due to throttling through the small set of holes at the buffer end of the recuperator cylinder is exceedingly small.

Therefore, considering the equilibrium of the floating piston and neglecting the drop of pressure through the valves, we have

$$pA_b - p(A_b - a_b') + p_a'A_a' - p_aA_a - R_f = 0$$

$$pa_b + p_a'A_a' + p_a'A_a' - R_f = 0$$

$$\text{hence } p_a' = \frac{p_aA_a + R_f - pa_b}{A_a'} = (p\frac{A_a}{A_a'} + \frac{R_f}{A_a'}) - p\frac{a_b}{A_a'}$$

The pressure in the hydraulic or recoil cylinder becomes,

$$p = Sh_o + \frac{K_o^2(A_a - a_b)^2A^2V^2}{175\,A_a^2w_o^2} + p\frac{A_a}{A_a'} + \frac{R_f}{A_a'} - p\frac{a_b}{A_a'}$$

therefore

$$p(1 + \frac{a_b}{A_a'}) = Sh_o + \frac{K_o^2A_a'^2A^2V^2}{175A_a^2w_o^2} + p_a'' \quad \text{where } p_a'' = p_a\frac{A_a}{A_a'} + \frac{R_f}{A_a'}$$

The resistance to recoil, becomes, since

$$(1+\frac{a_b}{A'_a})=\frac{A_a}{A'_a}$$

$$K = \frac{Sh_o A'_a A}{A_a} + \frac{K_o^2 A'^3_a A^3 V^2}{175 A^3_a w_o^2} + p''_a \frac{A'_a}{A_a} + \Sigma R - W_r \sin\phi$$

or in terms of p_a,

$$K = p_a A + \frac{K_o A'^3_a A^3 V^2}{175 A^3_a w_o^2} + \frac{Sh_o A'_a A}{A_a} + \frac{R_f A}{A_a} + \Sigma R - W_r \sin\phi$$

where $\Sigma R = R_s + R_p + R_g$

R_s = stuffing box friction of piston rod.

R_p = hydraulic piston friction

R_g = guide friction

As will be shown in the theory of packing the stuffing box and hydraulic piston frictions depend upon the pressure in the hydraulic cylinder.

Further a recoil buffer action is obtained towards the end of the recoil by making w_o variable, i.e. substituting w_x in the above equation for w_o.

The total resistance to recoil may be simplified into the following equation:

$$K = C_1 p_a + C_2 \frac{V^2}{w_o^2} + (\Sigma R + C_3) - W_4 \sin\phi$$

Towards the end of recoil w_o changes to w_x

$$C_1 = A = C_2 = \frac{K_o A'^3_a A^3}{175\ A^3_a} \quad \text{where } K_o = \frac{1}{0.9}$$

$$C_3 = \frac{Sh_o A'_a A}{A_a} + \frac{R_f A}{A_a}; \quad R = R_s + R_p + R_g$$

The above equation is exactly equivalent to previous recoil equations with the fundamental difference that now we have a constant orifice during the greater part of recoil in place of the previous variable orifice.

If the term $C_2 \frac{V^2}{w_o^2}$ is large as compared with the other terms in the

equation for the resistance to recoil, K obviously varies considerably with the velocity of recoil, having a large peak at maximum velocity. To reduce this peak in the resistance curve, we must have a larger orifice with less throttling and a consequent longer recoil together with a higher air pressure than would be used on other types of recoil systems. Hence, we may completely limit the ratio of the peak to the average resistance to recoil by suitable high air pressure, a large constant orifice and a long recoil. Thus we see the limitation of this system for high elevation and consequqnt short recoil.

THEORY OF PACKING. To prevent leakage of air or oil under high pressures in the cylinders the packing is designed to be subjected to the actual intensity of pressure of the air or oil as the case may be. The packing, however, cannot be regarded as subjected exactly to a hydraulic pressure since the pressure in one direction differs from the pressure in a direction at right angles by the amount of the previous pressure multiplied by Poisson Ratio. To compensate for the deficiency in oil or air pressure normal to the cylinder surface or piston rod, packing springs are introduced. These springs are designed for a slightly excess pressure normal to the cylinder surface in addition to compensating for the air or oil hydrostatic pressure. The total normal force multiplied by the coefficient of friction of the packing gives the total friction. Thus we see the friction may be considered as consisting of two components; the friction component due to the oil or air, and that due to the packing springs. The packing frictions, therefore, are a linear function of the pressure and may be represented by an equation as follows: $R = c_1 p + c_2$

a_1 = silver contact of outer flange

c_1 = gross length of outer flaps of flange

K = Poissons' ratio for leather = 0.73

p_a = intensity of air pressure

p_a' = intensity of oil side of floating piston

p_o = intensity of pressure in leather on air side caused by packing springs.

p_o' = intensity of pressure in leather on oil side caused by packing springs.

p_q = intensity of pressure in grease caused by reaction of spiral spring on slide moving on inner rod of floating piston.

R_s = reaction of spiral spring on grease slide at assembled height.

R_b = reaction of belleville washers on annular area of packing on air side at assembled height.

R_b' = reaction of belleville washers on oil side of packing at assembled height.

f = coefficient of friction of leather = .05

f_1 = coefficient of silver = .09

Then in the metric system of units if all lengths or dimensions are measured in millimeters and pressures in kilograms per sq. cm. and spring reaction in kilograms,

$$p_q = \frac{400\ R_s}{\pi(d^2-d_n^2)} \qquad p_o = \frac{400\ R_b}{\pi(d^2-d_n^2)}$$

$$p' = \frac{400 R_b'}{\pi(d^2-d_n^2)}$$ where $\frac{\pi}{4}(d^2-d^2)$ is the annular of the packing ring in sq. mm. and the total pressure in the packing, become,

Grease pressure = p_a+p_q (considered a purely hydrostatic pressure)

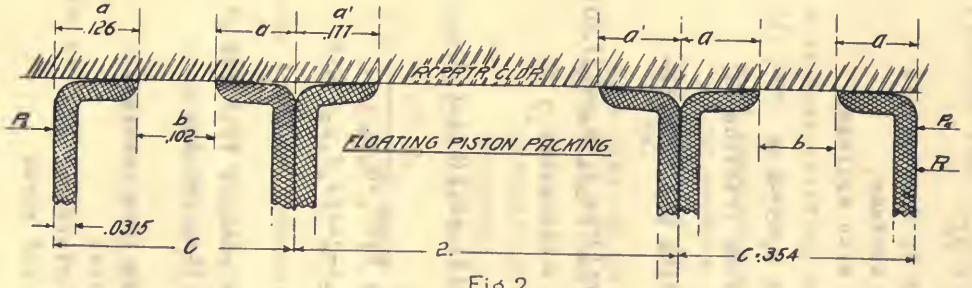


Fig.2

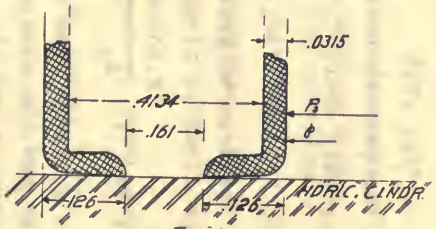


Fig.4

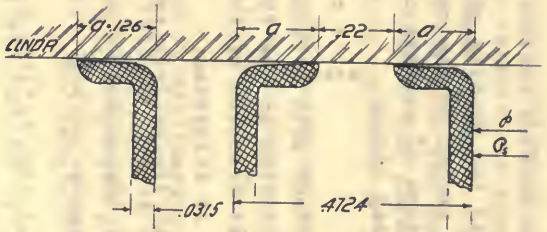


Fig.3

PISTON PACKING

Air side packing pressure = $p_a + p_o$ Kg per sq. cm.

Oil side packing pressure = $p'_a + p'_o$

If we multiply the packing pressures by K we obtain the pressure normal to the cylinder surface. From the diagram of packing contact with cylinder fig. (2) we may assume

$$\frac{a}{2} = c-2e \text{ and } a_1 = c'-2e$$

since the contact with cylinder is some fraction of the breadth of flap of the flange.

The total friction of the floating piston packing becomes, in kilograms,

$$R_f = \pi d[(bf+af_1)(p_o+p'_o)+(bf+af_1)(p_a+p'_a)+2a_1f_1(p_a+p_q)].01K$$

$$= \pi d[bf+af_1)(p_o+p'_o)+2a_1f_1p_q].01K + \pi d[(bf+af_1)(p_a+p'_a)+2a_1f_1p_a].01K$$

$$= \pi d[(bf+af_1)(p_o+p'_o)+2a_1f_1p_q] .01K+ \pi d[2(bf+af_1)+a_1f_1]p_a .01K$$

Thus the packing friction reduces to $R_f=C_1+C_2p_a$ (kilograms) where $C_1 = \pi d[(bf+af_1)(p_o+p'_o=+2a_1f_1p_q].01K$

and $C_2 = \pi d[2(bf+af_1)+2\ a_1f_1] .01K$

The drop of pressure due to the packing friction of the floating piston, becomes, in lbs. per sq. inch

$\pm (p'_a-p_a)=P_f= \dfrac{2.205-R_f}{A_a}$ very roughly (See previous exact formula)

Where A_a = the area of the floating piston in sq. inches and the plus sign for recoil and minus for counter recoil.

The above formulae are not strictly exact but give however, a very close approximation, indeed, of the friction drop due to the floating piston.

In the design of the packing given later, tables are shown for proper width of packing and excess pressures due to springs as well as a table for

Belleville given in the appendix.

STUFFING BOX FRICTION. In the stuffing box packing the leather and flaps of the packing flanges, come in contact with the peripheral surface of the piston rod.

A typical stuffing box packing is shown in figure (3). If, now, as before

p_o = intensity of pressure caused by bellevilles or packing springs in Kg. per sq. cm.

p = pressure in hydraulic cylinder Kg. per sq. cm.

d_r = diam. of piston rod in millimeters.

d_o = outer diameter of packing ring in millimeters

R_b = Belleville reaction at assembled load on annular area of packing ring measured in kilograms

Then in metric units as before,

$$p = \frac{400\ R_b}{\pi(d_o^2-d_r^2)} \text{ where } \frac{\pi}{4}(d_o^2-d_r^2) \text{ is the annular area of the packing ring and the stuffing box friction, becomes,}$$

$$R_s = \pi d[(bf+af_1)(p_o+p)+a_1f_1p].01K \text{ Kilograms}$$

$$= \pi d(bfp+afp+a_1f_1p+bp_of+ap_of_1).01K$$

$$= \pi d[p(bf+af+a_1f_1)+p_o(bf+af_1)]\ .01K$$

Or as before $R_s=C_1+C_2p$ where $C_1 = \pi\ dp_o(bf+af_1).01K$

and $C_2 = \pi d(bf+af+a_1f_1).01K$

HYDRAULIC PISTON FRICTION. The packing in the hydraulic piston is similar to that in the floating piston. We have, if

d = the diam. of the hydraulic cylinder in mm.

d_1 = the inner diameter of the packing ring, in mm.

R_b = the reaction of the belleville washers on the packing, in Kg.

$$p_o = \frac{400\ R_b}{\pi(d^2-d_1^2)}$$

or in the form $R_p = C_1+C_2p$

$$C_1 = \pi d p_o (bf+af_1).01K$$

$$C_2 = \pi d (bf+af_1+ a_1f_1)\ .01K$$

CALCULATED RETARDED VEIOCITY CURVE OF A TYPICAL SYSTEM.

During the powder period, we have

$$V_f - \frac{Kt}{m_r} = V$$

or in terms of any two given points,

$$V_{fm}-V_{fn} - \frac{K(t_2-t_1)}{m_r} = V_m-V_n$$

During the retardation, we have $V = \frac{-Kt}{m_r}$

$$\therefore\ V_n-V_m = \frac{K(t_m-t_n)}{m_r}$$

The corresponding displacement during the interval, becomes,

$$\frac{V_m+V_n}{2}\ (t_m-t_n)= X_{mn}= \Delta\ X_1$$

where

$$K = \frac{Sh_oA_a'A}{A_a} + \frac{K_o^2A_a'^3A^3V^2}{175\ A_a^3w_o^2} + p_a'' \frac{A_a'}{A_a} + ER - W_r \sin \emptyset$$

and $p_a'' = p_a\frac{A_a}{A}+\frac{R_f}{A_a'}$

Friction Values:

(1) Floating piston friction

$K = 0.73$ $R_s = 950$ lbs. $R_B = 2150$ lbs.

$R_b' = 4080$ lb. $p_g = 190$ lbs. per sq. in. (calculated)

$p_o = 880$ lbs. per sq.in. $p_o' = 1690$ lbs. per sq.in.

Hence from the previous theory on packing, the floating piston friction may be expressed in the following formula: $R_f = .53\ p_a + 420$ lbs.

(2) Hydraulic piston packing friction

$R_b = 2980$ $P_o = 2080$ lbs. per sq.in.

Hence $R_p = .121\ p + 280$

(3) Stuffing box friction

$R_b = 2810$ lbs. $p_o = 1970$ lbs. per sq.in.

Hence $R_s = .063\ p + 140$

(4) Guide friction neglected

From (2) and (3) we obtain for the total friction resistance during the recoil, $\Sigma R = .184\ p + 420$

Drop of pressure through check valve:

$a_s = 0.0779$ sq.in. $Sh_o = 13$ lbs.

Hence $P = \frac{13}{0.0779} = 167$ lbs. Use 150 lbs.

Air pressure curve:

$A = 1.314$

U_i = initial air vol. = 188.7 cu. in.

p_{ai} = initial air pressure = 1675 lbs. per sq. in.

$$p_{af} = p_{ai}\left(\frac{U_i}{U_i - Ab}\right)1.3 = 1675\left(\frac{188.7}{188.7 - 1.314b}\right)1.3$$

$$= 2731 \text{ lbs. per sq. in.}$$

where b = length of recoil in inches.

TABLE OF AIR PRESSURES:

$$p_a = p_{ai}\left(\frac{U_i}{U_i - Ax}\right) = 1675\left(\frac{188.7}{188.7 - 1.314x}\right)1.3$$

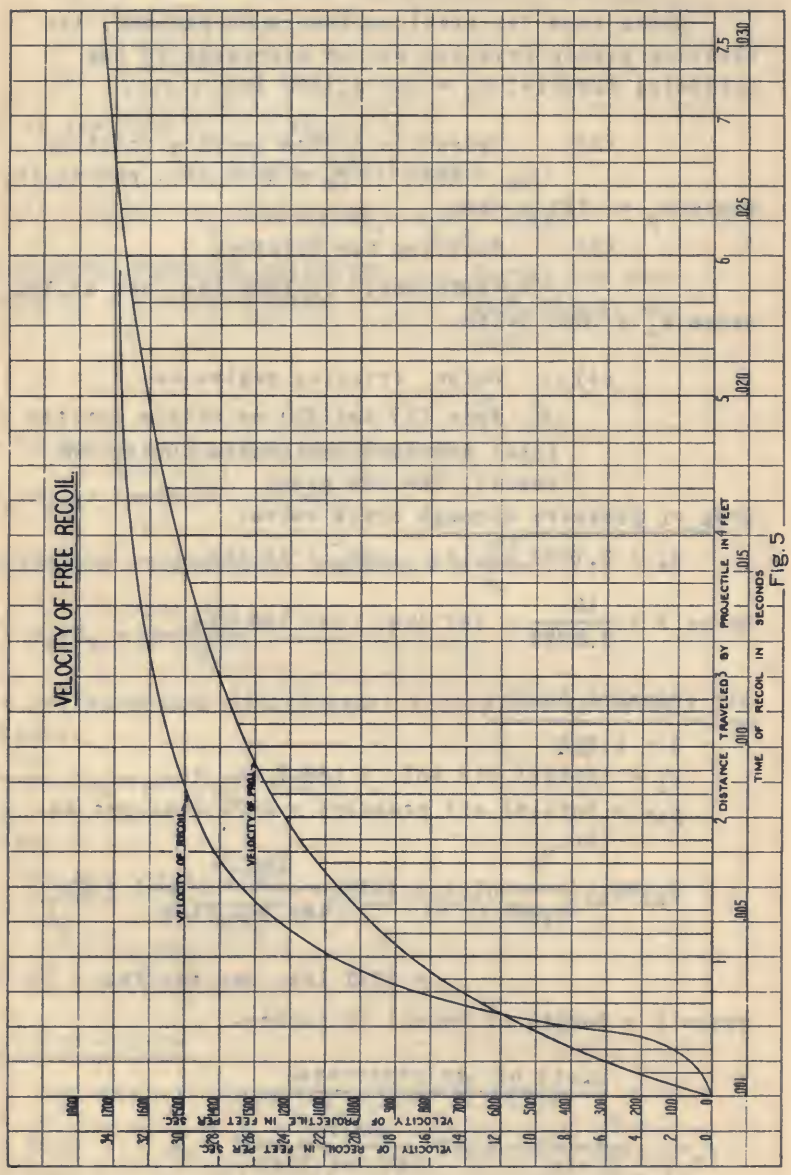

VELOCITY OF FREE RECOIL
VELOCITY OF RECOIL
VELOCITY OF FULL
VELOCITY OF RECOIL IN FEET PER SEC.
VELOCITY OF PROJECTILE IN FEET PER SEC.
DISTANCE TRAVELED BY PROJECTILE IN FEET
TIME OF RECOIL IN SECONDS

Fig. 5

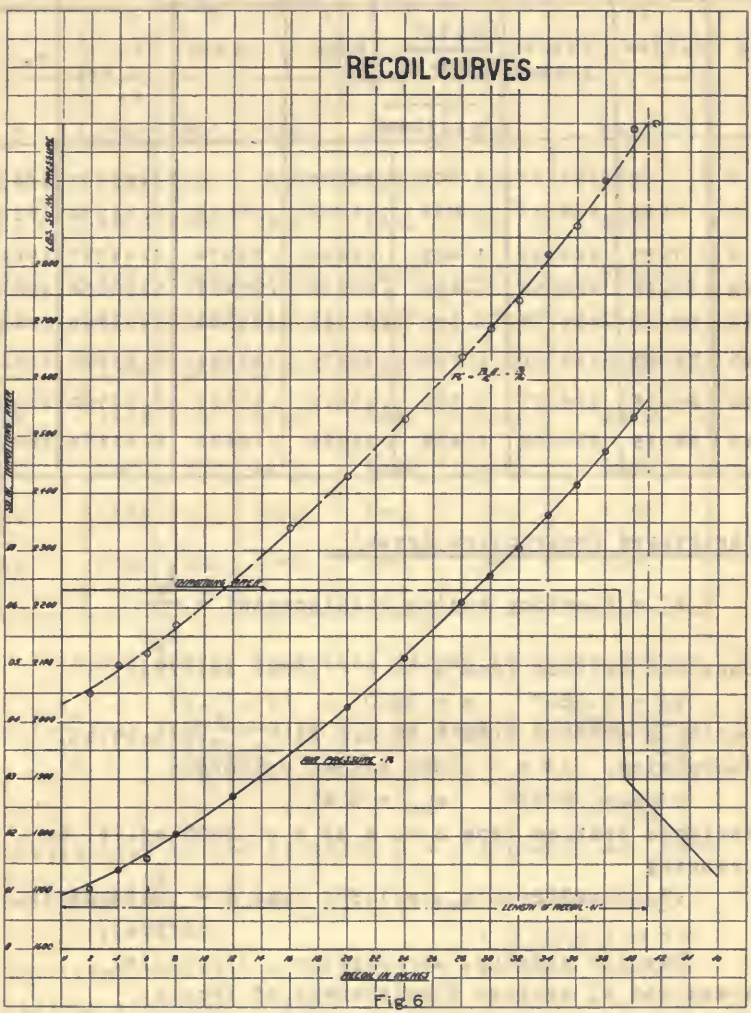


Fig. 6

a	b	c	d	e	f	g	h
x	1.314x	188.7 / 1.314x	182.7 / 188.7 − 1.314x	log d	1.3×c	laj 1.675 tf	p_a
0	0	188.70	1.000	.00000		3.22401	1675
2	2.63	186.07	1.014	.00604	.00785	3.23192	1706
6	7.88	180.82	1.043	.01828	.02376	3.24777	1769
14	18.40	170.30	1.105	.04336	.05637	3.28038	1907
22	28.91	159.79	1.180	.07188	.09344	3.31745	2077
30	39.42	149.28	1.264	.10175	.13228	3.35629	2271
38	49.93	138.77	1.359	.13322	.17319	3.39720	2496
42	55.15	133.55	1.412	.14983	.19478	3.41879	2623

Throttling Contraction Areas:

x' = floating piston displacement = $\frac{A_x}{A_a}$

Constant orifice from a to b = .0631 sq. in.

$x' = 9.685"$ $x = 39.0"$

Variable orifice from b to c ; at b = .0631 sq.in. decreasing, at c = .0302 sq.in. uniformly

$x'_{bc} = .0787"$ $x_{bc} = 0.4"$

Variable orifice from c to d at c = .0302 sq.in. decreasing

$x'_{cd} = .3958$ $x_{cd} = 1.6"$ at d = .0184 sq.in. uniformly

Diagram showing, air pressure, throttling areas and p''_a against displacement of recoil. See fig. (6).

Table pf p''_a

Recoil	P_a	$\frac{A}{A_a}P_a$	$.53P_a$	R_f	$\frac{R_f}{A'_o}$	P''_a
2	1706	1800	850	1270	250	2050
6	1760	1860	880	1300	260	2120
12	1870	1980	930	1350	270	2250
20	2027	2145	1010	1430	290	2430
28	2210	2340	1100	1520	300	2640
32	2305	2440	1150	1570	310	2750
36	2416	2550	1210	1630	320	2870
40	2533	2700	1270	1690	340	3040
44	2665	2820	1330	1750	350	3170
46	2730	2890	1360	1780	360	3250

Substituting numerical values in the equation,

$$p\left(1+\frac{a_b}{A'_a}\right)=sh_o+\frac{K_o^2A_a'^2A^2V^2}{175A_a^2w_o^2}+p''_a,$$ we have $$p = 140+.01\frac{V^2}{w_o^2}+1.067p''_a$$

Substituting in the equation $R = .184p+420.$ we have $$R=.001845\frac{V^2}{w_o^2}+0.1968\,p''_a+443.$$ Substituting R in the equation for the total resistance to recoil K, we have, at the maximum elevation = 19°.

$$\frac{K}{m_r}=\frac{465.57}{10^6}\frac{V^2}{w_o^2}+.043\,p''_a+8.58.$$ Then substituting in the dynamic equation, we have $V_{fm}-V_{fn}-(0.116V^2+.043p''_a+8.58)$

$t = V_m - V_n$ where $V^2 = \frac{(V_m+V_n)^2}{2}$ and $\frac{V_m+V_n}{2}$ $t = x_m - x_n$

On the bases of these equations the following table is made for the restrained velocity and displacement during the powder period and subsequent retardation until the variable orifice at the end of recoil is reached.

It is to be noted that after the powder period (in the above equation) $V_{fm} - V_{fn} = 0$.

Retarded velocity curve during powder period,

El= 19°

$a = .116V^2$

$b = .043P''_a$

$c = .116V^2 + .043p''_a + 8.58$

$$V_f = \sqrt{\frac{.116}{a}V^2 + \frac{.043P''_a}{b} + .58} \qquad t\ V_{t_2} V_{t_1}$$

Recoil	a	b	c	ct	V_f	V_t	$V_1' + V_2'$	t	2t	x	2x
1.33	59	78	146	.29	3.65	27.52	25.84	002	007		.1105
2.36	78	78	165	.49	2.45	29.48	28.50	003	010		.1963
3.80	94	79	182	.73	1.77	30.52	30.00	004	014		.3168
4.90	104	80	193	.58	.72	30.66	30.59	003	017		.4081
5.27	108	81	198	.20	.37	30.83	30.75	001	018		.4389
5.64	110	81	200	.20	.21	30.84	30.84	001	019		.4697

Retarded velocity curve during retardation period with constant orifice Cl.= 19°.

After powder pressure (constant orifice)

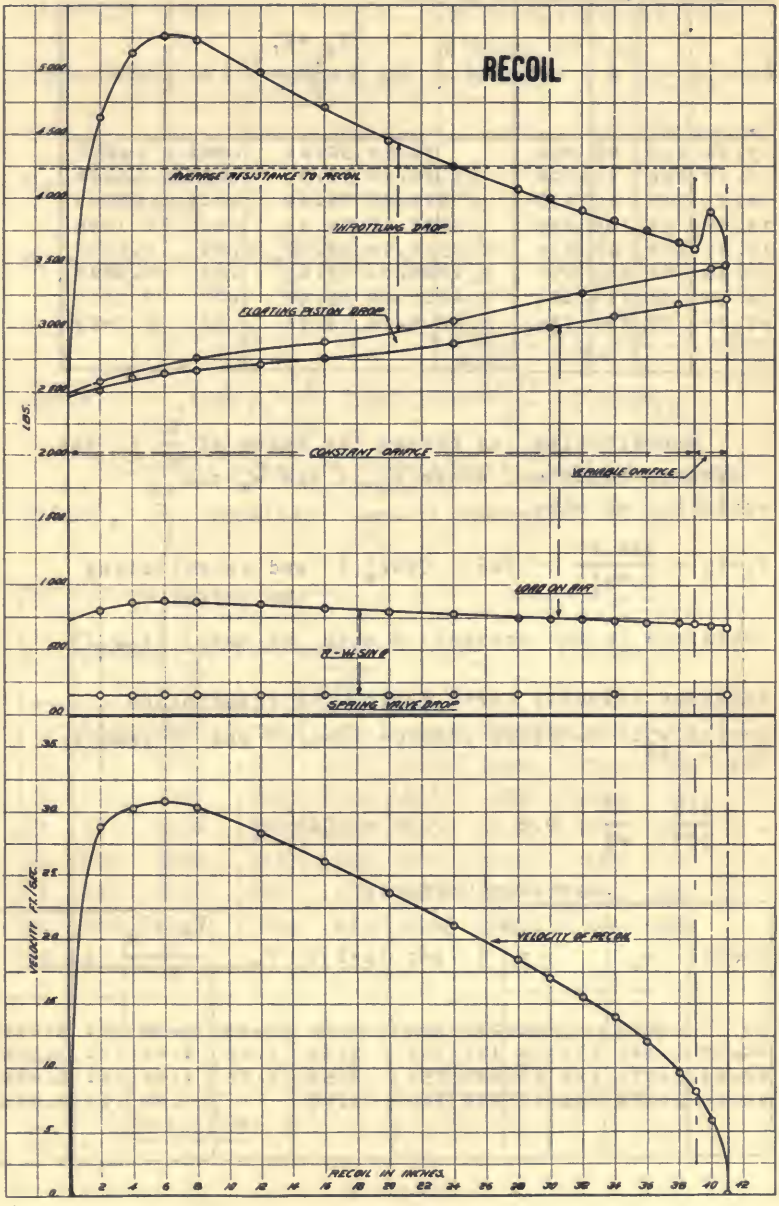

RECOIL
AVERAGE RESISTANCE TO RECOIL
INPUTTUBE DROP
FLOATING PISTON DROP
CONSTANT ORIFICE
VARIABLE ORIFICE
LOAD ON RAM
RECOIL VALVE
SPRING VALVE DROP
VELOCITY OF RECOIL
LBS.
VELOCITY FT./SEC.
RECOIL IN INCHES.

Fig. 7

Recoil	a	b	c	cd	V_r	$\frac{V_{r_2}+V_{r_1}}{2}$	t	2x
5.64	110	81	200		30.84	30.84	.001	.4697
6.46	110	82	202	.2	30.44	30.54	.001	.5376
8.61	106	82	197	.79	29.25	29.65	.004	.7167
13.98	96	105	190	1.90	26.52	27.47	.01	1.1635
19.89	76	90	175	1.75	22.94	23.82	.01	1.6573
27.30	66	161	161	3.22	18.04	19.65	.02	2.2713
34.50	40	144	144	2.88	12.08	13.52	.02	2.8717
39.00	22	134	134	2.68	6.64	7.98	.02	3.2453

Substituting, as before the value of $\frac{K}{m_r}$ in the dynamic equation, where w_o is now w_x and variable, we have,

$$V_n - V_m = \frac{466.V^2}{10^6 w_x^2} + 858 + .043 P_a'' t$$ and substituting the values w_x obtained in the preceeding work, we have, (fig.7)

Retarded velocity curve during the retardation period with variable orifice towards end of recoil, Cl. = 19°

$$a = \frac{466}{10^6} \frac{V^2}{w_x^2} + 9.6 \qquad b = .043\ P_a''$$

Variable Orifice.

Recoil	w_o		a	b	a+b	(a+b)t	V_r	$\frac{V_{r_1}+V_2}{2}$	t	2x
39	.0631		22	112	134	2.68	6.64	7.98	.02	3.2453
40.22	.0295	870	28	115	143	1.43	3.92	4.64	.01	3.3517
40.84	.0270	228	16	117	133	1.33	1.23	1.90	.01	3.4031
40.95	.0263	692	12	118	130	1.30		.92	.01	3.4123

CALCULATED PRESSURES AND PRESSURE DROPS.

The total resistance to recoil may be expressed by the dynamic equation: $pA+E-W_r \sin \phi = K$ where $p = sh_o+P_o+P_f+p_a$

$$P_o = \frac{K_o^2 A_a'^2 A^2 V^2}{175 A_a^2 w_o^2}$$

$$P_f = p_a'-p_a = \frac{p_a A_a}{A_a'}+\frac{R_f}{A_a'}\frac{a_b}{A_a'}p - p_a = (\frac{A_a}{A_a'}-1)p+\frac{R_f}{A_a'}-\frac{a_b}{}p$$

$$=(\frac{5.3}{4.97}-1)p_a+\frac{R_f}{A_a'}-.0667p$$

hence $P_f = .065\ p_a+\frac{R_f}{A_a'}.0667(140+\frac{.01V^2}{w_o^2}+1.0667p_a''$

Table for pressure drop across floating piston:

$$2.065p_a+\frac{R_f}{A_a'}.0667(140+\frac{.01V^2}{w_o^2}+1.0667p_a'')$$

x	$.065p_a$	$\frac{R}{Aa'}$	V^2	$\frac{.01V^2}{w_o^2}$	$1.06P_a''$		.0667	
2	109	250	762	1920	2190	4250	280	70
4	113	260	912	2300	2240	4680	310	80
8	117	260	918	2320	2320	4780	320	90
16	126	280	681	1720	2500	4300	290	120
24	137	290	445	1120	2700	3960	260	170
32	149	310	240	600	2930	3670	240	220
40	164	340	34	414	3250	3800	250	250

In terms of the intensity of pressure, we have $\frac{K}{A} = p + \frac{R}{A} - \frac{W_r \sin\phi}{A} = \frac{R-W_r\sin\phi}{A} Sh_o+P_f+p_o+p_a$

Let $a = \frac{R-W_r\sin\phi}{A}$

$b = a+ Sh_o$ $\quad$ $d = c+P_f$

$c = b+p_a$ $\quad$ $e = d+P_o$

From these values the following table was computed for the various pressure drops.

Recoil	$\frac{RW_r \sin\phi}{A}$	$a+sh_o$	$b+P_a$	$c+P_f$	$d+P_o$
2	650	800	2510	2580	4630
4	720	870	2610	2680	5130
6	730	880	2540	2720	5270
8	720	870	2670	2760	5230
12	700	850	2720	2830	4980
16	670	820	2760	2880	4710
20	640	790	2820	2950	4450
24	620	770	2880	3050	4250
28	590	740	2950	3160	4070
30	590	740	3000	3220	4000
32	580	730	3040	3040	3910
34	570	720	3080	3310	3830
36	560	710	3130	3360	3750
38	560	710	3180	3410	3660
39	550	700	3210	3430	3610
40	530	680	3210	3460	3900
41	520	670	3240	3480	3480

u'' = travel of shot in the bore 89.87

$V_n = \frac{an}{b+n} = \frac{2046 \times 7.5}{1.377+7.5} = \frac{15340}{8.877}$ 1730

V_f 33.8

$V_o = \frac{V_m(W+.5\bar{w})}{W_r} = \frac{1730 \times 16.75}{1050}$ 27.6

$t_1 = \frac{s}{2} \frac{u''}{12V_m} = \frac{s}{2} \frac{7.5}{1730} = \frac{11.22}{1730} =$ .0065

P_m = max. pressure 36000 lbs.

$P_b = 1.12P_m$ 40300 lbs.

P_e = mean constant pressure of breech

$$\frac{12WV_m^2}{64.4u''s} = \frac{15.95\times\overline{1730}^2}{64.4\times7.5\times6.95} = 14200$$

$$e = 89.87[(\frac{27}{16}\times\frac{36000}{14220} - 1) \pm \sqrt{(1 - \frac{27}{16}\frac{36000}{14220})^2 - 1}]$$

$$89.87(3.2722 \pm \sqrt{9.707}) = 14.08$$

$$P_{ob} = \frac{27}{4} c^2 \frac{u''}{(c+u'')^3} P_b = \frac{27}{4} \times \frac{\overline{14.08}^2 \times 89.87 \times 40300}{(103.85)^3}$$

$$4315$$

$$t_2 = \frac{2(V_g - V_o W_r)}{P_{ob} \times s'' \times g} = \frac{2(33.8-27.6)1050}{4315\times6.95\times32.2} = \frac{2\times6.2\times1.050}{4.315\times6.95\times32.2} =$$

$$.01248$$

$T = t_1 + t_2 = .0065 + 0135 \quad .02$ seconds

CALCULATION AND COMPARISON OF AVERAGE RESISTANCE.

$$x_1 = \frac{u''(w+0.5\bar{w})}{12(W_r+W+\bar{w})} = \frac{89.87\times16.75}{12\times1067.53} = .1175$$

$$x_2 = \frac{S''P_{ob}t_2^2}{W_r g \times 3} + V_o t_2 = \frac{6.95\times4.315\times\overline{01348}^2}{1.050 \times 32.2 \times 3} + 27.6\times.01348$$

$$= .00005 + .372$$
$$= .372$$

$E = .1175 + .372 = .49$

$T = .019$ second

$E = .50' = 6$

$$K = \frac{\frac{1}{2} mV_1^2}{b} = \frac{\frac{1}{2} 32.6\times\overline{33.8}^2}{3.4-.5+33.8\times.019} = 5300 \text{ (lbs)}$$

$$K = \frac{\frac{1}{2}mV_1^2}{b} = \frac{\frac{1}{2} 32.6 \times \overline{33.8}^2}{3.4} = 5500 \text{ (lbs)}$$

Average from the curve of forces 4230

A = 1.314 hence total force = 5530 lbs.

PEAK VALUES

Curve of resisting forces gives an average 5550 lbs. for K. Peak value of K from the curves 5280 × 1.314 = 6940 lbs.

Peak ratio $d = \frac{6940}{5530} = 1.25$ with respect av.recoil force from curve.

Peak ratio $d' = \frac{6940}{5300} = 1.30$ with respect computed av. recoil reaction.

RELATIVE SIZE OF ORIFICE UNDER DIAPHRAGM AND COUNTER RECOIL THROTTLING ORIFICE.

$Q_f = A_a x_f = 5.303\ x_f$

$Q_h = A \times 41 = 1.314 \times 41 = 53.9$

$x_f = \frac{53.9}{5.303} = 10.11$

A = .3945

$\frac{.7087-.6811}{2.7953} \times .35 = \frac{.0276}{2.7953} \times .35 = .00346$

$d_x = .00346+.6811=.6846$ in.

$A_x = .366$

Smallest area under the diagram . 3945 - .366 = .0285 sq.in.

Largest throttling area = .00665 sq.in.

∴ the throttling take place not under the diaphragm.

RELATIVE DISPLACEMENT OF FLOATING PISTON.

$A_a : A :: x_r : x_f$ where x_r = displacement of recoiling parts

x_f = displacement of floating piston

$$x_f = \frac{1.314}{5.303} x_r$$

Variable throttling begins $= x_f = 5.3545$ on recoil

corresponding $x_r = \frac{5.3028 \times 5.156}{1.314} = 20.80$

take .305 in.

Total floating piston displacement $= x_f = \frac{1.314}{5.303}$

$\times 41 = 10.1595$

Variable orifice ends at 41-20.5=20.5 of counter recoil

THROTTLING AREA.

Distance	Depth of groove	Area of one gun	Total Orifice	Recoil Disp.	Recoil in inches
5.9055	.0469	.003325	.00665		20.50
5.3150	.0453	.003212	.00642	2.38	22.88
4.5276	.0421	.002985	.00597	5.56	26.06
3.7402	.0386	.002737	.00547	8.74	29.24
2.9528	.0339	.002404	.00482	11.92	32.42
2.5591	.0307	.002177	.00435	13.50	34.00
2.3622	.0287	.002035	.00407	14.30	34.80
2.1654	.0272	.001928	.00386	15.09	35.59
1.9685	.0248	.001758	.00352	15.89	36.39
1.7717	.0220	.001560	.00312	16.68	37.18
1.5748	.0193	.001368	.00274	17.48	37.98
1.3780	.0157	.001113	.00223	18.27	38.77
.9843	.0063	.0004467	.00089	19.82	40.32
.5906	.0059	.000413	.00083	21.45	42.05
.0					

$P'_{a'}$

$P'_{a'} = P_a$ - Floating piston friction

$P'_{a'} = P_a - R_f$

$R_f = .53\ P_a + 420$

C'Recoil	P_a	R_f	$P'_{a'}$
0	2570	340	2,230
1	2530	340	2,290
2	2510	330	2,180
3	2480	330	2,150
6	2390	320	2,070
12	2240	300	1,940
20	2050	290	1,760
28	1890	270	1,620
34	1790	260	1,530
37	1740	260	1,480
38	1730	260	1,470
39	1720	250	1,470
40	1710	250	1,460
41	1700	250	1,450

VELOCITY OF COUNTER RECOIL.

$A_a V'_f = AV_r$ where V'_f = velocity of floating piston

V_r = velocity of recoiling parts.

$V'_f = \frac{A}{A_a} V_r$

A = quantity of oil throttled $= (A'_a - A_b + a_b) V'_f$

$$P'_a = \frac{K''a^2}{W_x^2} = P = \text{hydraulic piston pull (where } P'_a - P_a - \frac{R_f}{A'_a})$$

$$PA - R = M_r V_r \frac{dV_r}{dx}$$

$$R = .1211\ p + 279 + .0634P + 138 = .1845p + 417$$

$$\frac{K''Q^2}{W_x^2} = \frac{[K(A'_a - A_b + a_b)\frac{A}{A_a}V_r]^2}{175W_x^2} = \frac{K'V^2}{W_x^2}$$

$$PA - .1845\ P - 417 = M_r V_r \frac{dV_r}{dx}$$

$$P(A - .1845) - 417 = M_r V_r \frac{dV_r}{dx}$$

$$(P'_a - \frac{K'V^2}{W_x^2})(A - .1845) - 417 = M_r V_r \frac{dV_r}{dx}$$

$$(P'_a - \frac{K'V^2}{W_x^2})(A - .1845)dx - 417\ dx = \frac{1}{2} M(V_1^2 - V_2^2)$$

AVERAGE VELOCITY OF AN INTEGRAL PERIOD.

In computing the velocity of recoiling parts by the formula

$$P'_a A\ dx - \frac{KA(\frac{V_1 + V_2}{2})^2}{W_o^2} - R\ dx = \frac{1}{2} m_r (V_1^2 - v_2^2).$$ We can take for value of $(\frac{V_1 + V_2}{2})^2$ as it simplifies the equation to obtain V_2. It is more accurate than taking either V_1 or V_2 as the velocity for the integral period.

Its deviation from the actual average is very small. Proof

Let $V_1 = 2$ and $V_2 = 2.25$. Actual average velocity of the intervals $V_1 V_2$ =

$$\left(\frac{2+2.25}{2}\right)^2=\left(\frac{4.25}{2}\right)^2=(2.125)^2=4.5156$$

$$V_1^2=4$$

$$V_2^2=5.0625$$

$$\text{Approximate average}=\frac{V_1^2+V_2^2}{2}=\frac{9.0625}{2}=4.5312$$

$$\%\ \text{Deviation}\ \frac{4.5312-4.5156}{4.5156}\times 100=\frac{1.56}{4.5156}=0.34$$

COMPUTATION OF K.

$$\text{At max. velocity}\ \frac{dv}{dt}=0$$

$$pA=\Sigma R$$

$$p=P'_a-\frac{K_1V^2}{W_x^2}$$

$$\Sigma R=.1845\,p+417=1845\left(P'_a-\frac{K_1V^2}{W_x^2}\right)+417$$

$$\therefore\ P_aA-\frac{K_1V^2}{W_x^2}A=.1845\left(P'_a-\frac{K_1V^2}{W_x^2}\right)+417$$

$$\left(P'_a-\frac{K_1V^2}{W_x^2}\right)-\frac{.1845}{A}\left(P'_a-\frac{K_1V^2}{W_x^2}\right)=\frac{417}{A}$$

$$\left(P'_a-\frac{K_1V^2}{W_x^2}\right)=\frac{\frac{417}{A}}{1-\frac{.1845}{A}}=369$$

Assuming max. velocity occurs on 16" of counter recoil:

$$1850-370=\frac{K_1V^2}{W_x^2}=1480$$

$$\frac{\left[K\left(A'_a-A_b+a_b\right)\frac{A}{A_a}\right]^2V^2}{175W_x^2}=1480\qquad\frac{\left[K(4.9714-.7006+.3314)\frac{1.314}{5.303}\right]^2V^2}{175\times(.0065)^2}$$

$$\frac{(K\times 4.6022\times 1.314)^2\ \overline{3.28}^2}{175\ \times\ (.00665)^2\ \times\ \overline{5.303}^2} = 1480$$

$$K^2 = \frac{1480\times 175\times \overline{.00665}^2\ \times\ \overline{5.303}^2}{(4.6022\times 1.314\times 3.28)^2} = ?\quad K = .9048$$

Use K = 0.9

EQUATION OF FORCES ON COUNTER RECOIL.

$$\frac{K'V^2}{W_x^2} = \frac{[K'(A_a'-A_b+a_b)\frac{A}{A_a}V]^2}{175W_x^2} = \frac{[\frac{1}{9}(4.9714-.7006+.3314)\frac{1.314}{5.303}]^2V^2}{175\ \times\ W_x^2}$$

$$= \frac{(1.11\times 4.6022\times 1.314)^2V^2}{(13.24\times 5.303)^2W_x^2} = \frac{9159}{10^6}\ \frac{V^2}{W_x^2}$$

$$(P_a' - \frac{9159}{10^6}\ \frac{V^2}{W_x^2})1.1295\ dx - 417\ dx = \tfrac{1}{2}M(V_1^2-V_2^2)$$

$$(1.1295\ P_a' - \frac{10345}{10^6}\ \frac{V^2}{W_x^2} - 417)\frac{2dx}{m} = V_1^2-V_2^2$$

$$(.06929P_a' - \frac{634.6676}{10^6}\ \frac{V^2}{W_x^2} - 25.5828)dx = V_1^2-V_2^2$$

$$V^2 = \frac{V_1^2+V_2^2}{2}$$

$$V_2^2 - V_1^2 = (.06929P_a' - \frac{317.3313}{10^6}\ \frac{V^2}{W_x^2} - \frac{317.3313}{10^6W_x^2}\ V_2^2 - 25.5829)dx$$

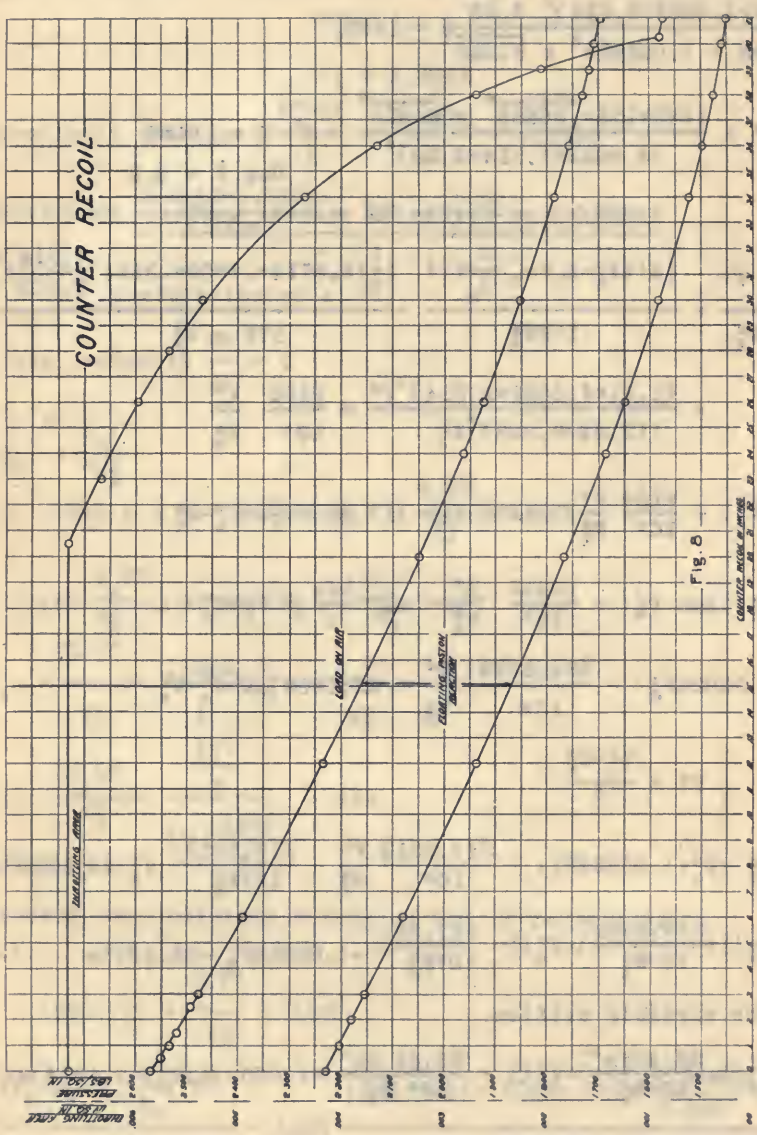
$$V_2^2(1+\frac{317.33dx}{10^6W_x^2}) = V_1^2(1-\frac{317.33}{10^6W_x^2}) + (.06939P_a' - 25.58)dx$$

For variable orifice.

$$V_2^2(1+\frac{26.44dx''}{10^6W_x^2}) = V^2(1 - \frac{26.44\ dx''}{10^6\ W_x^2})\times(.005774.P_a' - 2.1316)dx''$$

For constant orifice

$$V_2^2(1+.5979dx'') = V_1^2(1-.5979dx'') + (.005774P_a' - 2.1316)dx''$$


COUNTER RECOIL
Fig. 8

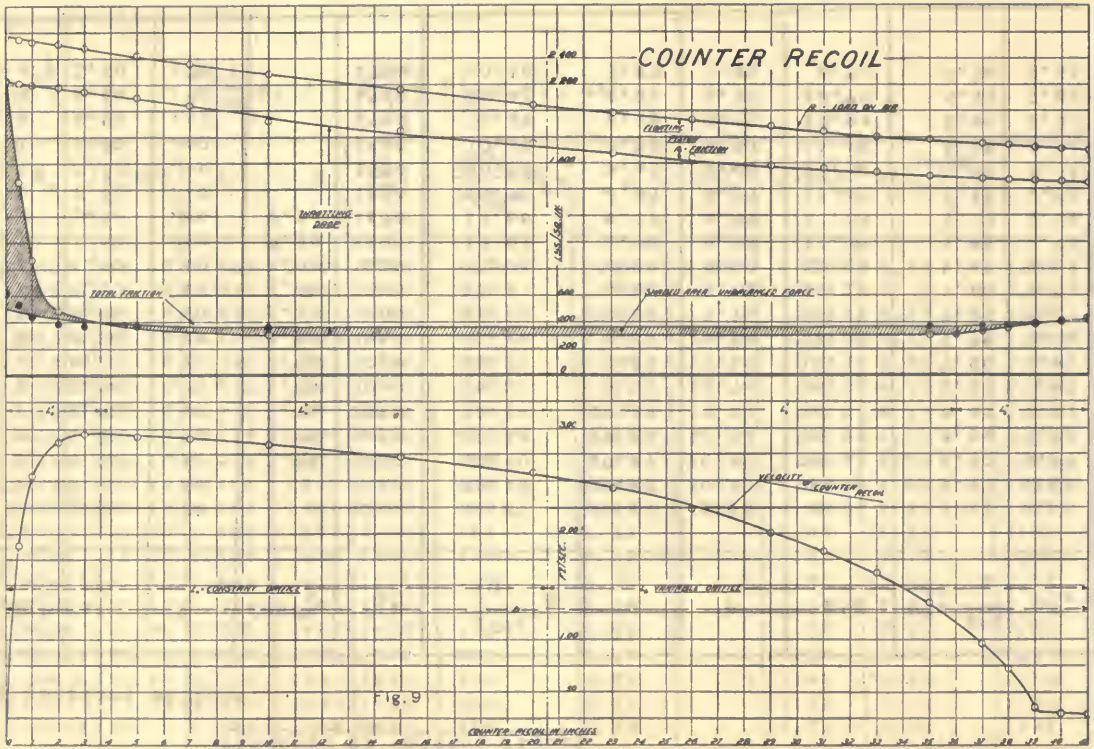

COUNTER RECOIL
LOAD ON AIR
FLOATING PISTON
FRICTION
THROTTLING DROP
TOTAL FRICTION
SHADED AREA UNBALANCED FORCE
VELOCITY OF COUNTER RECOIL
CONSTANT ORIFICE
VARIABLE ORIFICE
LBS/SQ.IN.
FT/SEC.
COUNTER RECOIL IN INCHES

Fig. 9

VELOCITY OF COUNTER RECOIL.

During constant orifice.

C'Recoil	dx'	a	b	$C=bV_1^2$	P_a',	$\frac{5774}{10^6}P_a'$,	d	e	c+e	$V_2^2=\frac{c+e}{a}$	V_r
5	5	1.30	.70	0	2220	12.80	10.67	5.34	5.34	4.11	2.03
1	1	1.60	.40		2210	12.76	10.63	10.63	10.63	6.65	2.58
2	1	1.60	.40	2.66	2180	12.60	10.47	10.47	13.13	8.20	2.86
3	1	1.60	.40	3.28	2150	12.44	10.31	10.31	13.59	8.50	2.92
4	1	1.60	.40	3.40	2130	12.30	10.17	10.17	13.57	8.48	2.91
6	2	2.20	.20	1.69	2080	12.02	9.89	19.78	18.09	8.23	2.87
8	2	2.20	.20	1.64	2030	11.72	9.59	19.18	17.54	7.98	2.83
10	2	2.20	.20	1.59	1980	11.44	9.31	18.62	17.03	7.75	2.78
11	1	1.60	.40	3.10	1960	11.32	9.19	9.19	12.29	7.67	2.77
12	1	1.60	.40	3.07	1940	11.21	9.08	9.08	12.15	7.60	2.75
13	1	1.60	.40	3.04	1910	11.03	8.90	8.90	11.94	7.47	2.73
14	1	1.60	.40	2.99	1890	10.91	8.78	8.78	11.77	7.55	2.71
15	1	1.60	.40	2.94	1870	10.80	8.67	8.67	11.61	7.26	2.69
16	1	1.60	.40		1850	10.70	8.57	8.57	11.47	7.17	2.68
17	1	1.60	.40		1830	10.56	8.45	8.43	11.30	7.07	2.66
18	1	1.60	.40		1810	10.45	8.32	8.32	11.15	6.97	2.64
19	1	1.60	.40		1780	10.28	8.15	8.15	10.94	6.85	2.62
20	1	1.60	.40		1760	10.15	8.02	8.02	10.76	6.73	2.59
20.b	0.5	1.30	.70		1750	10.10	7.97	3.99	8.70	6.70	2.59

During variable orifice.

$$V_2^2\left(1+\frac{26.44}{10^6 W_x^2}dx''\right) = V_1^2\left(1-\frac{26.44}{10^6 W_x^2}dx''\right) + \left(\frac{5774}{10^6}P_a'-2.13\right)dx''$$

C'Recoil	dx"	W_x	$10^6 W_x^2$	a	b	P_a'	$\frac{5774}{10^6 dx''}P_a'$	e	bV_1^2	$e+bV_1^2$	$V_2^2=\frac{e+bV_1}{a}$	V_r
21	.5	.0060	43.56	1.303	.697	1740	501	3.96	4.67	8.63	6.62	2.57
22	1.	.0065	42.25	1.626	.374	1720	994	7.81		10.29	6.43	2.54
23	1.	.00625	39.06	1.677	.323	1690	976	7.63		9.67	5.77	2.40
24	1.	.00612	37.45	1.706	.294	1670	965	7.52		9.21	5.40	2.32
25	1.	.00596	35.52	1.745	.255	1650	953	7.40		8.77	5.02	2.24
26	1.	.00583	33.99	1.778	.222	1630	942	7.29		8.41	4.73	2.18
27	1.	.00567	32.15	1.823	.177	1610	930	7.17		8.01	4.38	2.09
28	1.	.0055	30.25	1.873	.127	1600	924	7.11		7.67	4.09	2.02
29	1.	.00532	29.30	1.935	.075	1580	913	7.00		7.31	3.78	1.94
29.5	5.	.00522	27.25	1.485	.515	1576	455	3.48		5.43	3.65	1.91
30.	5.	.00512	26.21	1.504	.496	1568	453	3.47		5.28	3.51	1.87
30.5	5.	.00500	25.00	1.529	.471	1560	451	3.44		5.09	3.33	1.83
31	5.	.0049	24.01	1.551	.449	1550	448	3.42		4.91	3.16	1.78
31.5	5.	.00476	22.66	1.583	.417	1545	447	3.40		4.72	2.98	1.73
32.	5.	.00464	21.53	1.614	.386	1538	444	3.38		4.53	2.81	1.68
32.5	5.	.0045	20.25	1.653	.347	1530	442	3.35		4.33	2.62	1.62
33.	5.	.00436	19.01	1.696	.304	1522	440	3.34		4.14	2.44	1.56
33.5	5.	.00420	17.64	1.750	.250	1518	439	3.32		3.93	2.24	1.50
33.5	5.	.00420	17.64	1.750	.250	1518	439	3.32		3.93	2.24	1.50
34.	5.	.00403	16.24	1.815	.185	1510	436	3.30		3.71	2.04	1.43

$$V_2^2\left(1+\frac{26.44}{10^6 W_x}\,dx''\right)=V_1^2\left(1-\frac{26.44}{10^6 W_x^2}\,dx''\right)+(.005774P'_a-2.1316)dx''$$

C'Re-coil	dx"	W_x	$W_x 10^6$	a	b	P'_a	$\frac{5774}{10^6 dx''}P'_a$	e	$c+bV_1^2$	$V_2^2=\frac{e+bV_1^2}{a}$	V_r
34.5	5.	.00385	14.82	1.892	,108	1.505	4.35	3.28	3.50	1.85	1.36
35.	5.	.00365	13.32	1.992	.008	1.500	4.33	3.27	3.28	1.65	1.29
35.5	5.	.00345	11.90	2.111	.111	1.495	4.31	3.24	3.07	1.45	1.20
36.0	5.	.00320	10.24	2.290	.290	1.490	4.29	3.22	2.77	1.21	1.10
36.50	5.	.00295	8.70	2.520	.52	1.482	4.27	3.20	2.57	1.02	1.01
37.	5.	.00270	7.29	2.814	.81	1.478	4.25	3.19	2.36	.84	.92
.25	.25	.00252	6.35	2.041	.04	1.474	2.128	1.595	1.559	.76	.87
.50	.25	.00235	5.52	2.120	.120	1.470	2.123	1.590	1.512	.71	.84
.75	.25	.00220	4.84	2.365	.365	1.469	2.120	1.587	1.327	.56	.75
38.00	.25	.00205	4.20	2.575	.575	1.468	2.118	1.585	1.263	.49	.70
.25	.25	.00185	3.42	2.932	.932	1.465	2.115	1.593	1.125	.38	.62
.50	.25	.00165	2.72	3.425	1.425	1.463	2.113	1.581	1.033	.30	.55
.75	.25	.00133	1.77	4.750	2.750	1.462	2.110	1.578	.747	.16	.40
39.00	.25	.00100	1.00	7.610	5.61	1.460	2.108	1.575	.693	.08	.32
.25	.25	.00095	.90	8.350	6.35	1.458	2.105	1.572	.994	.11	.31
.5	.25	.0009	.81	9.160	7.16	1.455	2.100	1.567	.717	.08	.30
.75	.25	.00087	.76	9.700	7.70	1.462	2.095	1.562	1.002	.10	.29
40.	.25	.00085	.72	10.175	8.18	1.450	2.093	1.560	.742	.07	.26

LOGARITHMIC METHOD

(Check on velocity of c'recoil)

$$F_x - W_r(\sin\phi - n\cos\phi) - \Sigma R - \frac{cV^2}{W_x^2} = mV\frac{dV}{dx}$$

$$F_x - R - \frac{cV^2}{W_x^2} = mV\frac{dV}{dx}$$

$$A - \frac{cV^2}{W_x^2} = mV\frac{dV}{dx}$$

$$\left(P'_a - \frac{9159}{10^6}\frac{V^2}{W_x^2}\right)1.1295 - 417 = mV\frac{dx}{dx}$$

$$1.1295P'_a - \frac{10345}{10^6}\frac{V^2}{W_x^2} - 417 = mV\frac{dx}{dx}$$

$$A = 1.1295\,P'_a - 417$$

$$c = \frac{10345}{10^6}$$

$$\log\left(A - \frac{cV_2^2}{W_x^2}\right) = \log\left(A - \frac{cV_1^2}{W_x^2}\right) - \frac{2c(x_2 - x_1)}{2.3m_r W_x^2}$$

$$\log\left(\frac{cV_2^2}{W_x^2} - A\right) - \log\left(\frac{cV_1^2}{W_x^2} - A\right) = \frac{2c(x_2 - x_1)}{2.3\,mW_x^2}$$

$$\log\frac{\left(\frac{cV_2^2}{W_x^2} - A\right)}{\left(\frac{cV_1^2}{W_x^2} - A\right)} = -\frac{2c(x_2 - x_1)}{2.3m\,W_x^2}$$

$$\frac{\frac{cV_2^2}{W_x^2} - A}{\frac{cV_1^2}{W_x^2} - A} = \frac{1}{\text{Antlg}\,\frac{2c(x_2 - x_1)}{2.3m\,W_x^2}}$$

$$\frac{cV_2^2}{W_x^2} - A = \frac{\frac{cV_1^2}{W_x^2} - A}{\text{Antlg}\ \frac{2c}{2.\ m\ W_x^2}(x_2 - x_1)}$$

$$V_2^2 = \left[\frac{\frac{cV_1^2}{W_x^2} - A}{\text{Antlg}\ \frac{2c(x_2 - x_1)}{2.3\ mW_x^2}} + A\right] \frac{1}{\frac{c}{W_x^2}}$$

$$\frac{2c(x_2 - x_1)}{2.3mW_x^2} = \frac{2 \times 10345}{2.3 \times 10^6 \times 32.6} \frac{dx}{W_x^2} = \frac{276}{10^6} \frac{dx}{W_x^2}$$

$$V_2^2 = \left(\frac{\frac{cV_1^2}{W_x^2} - A}{\text{Antlg}\ \frac{276}{10^6} \frac{dx}{W_x^2}} + A\right) \frac{1}{\frac{c}{W_x^2}}$$

$$A = 1.1295\ P_a' - 417$$

$$C = \frac{10345}{10^6} \quad \frac{C}{W_x^2} =$$

$$a = \frac{276}{10^6} \quad \frac{dx}{W_x^2}$$

$$b = \text{Antlg}\ a$$

C're-coil	P'_a,	1.1295P'_a,	A	W_x^2 10^6	$\frac{c}{W_x^2}$	Δx	276dx	a	b	f
35	1500	1694	1277							
36	1490	1683	1266	13.47	768	.0833	23.00	1.7075	50.99	49.99
36 1/2	1482	1674	1257	12.04	859	.0416	11.50	.95514	9.02	8.02
37	1478	1669	1252	10.50	985	.0416	11.50	1.09524	12.45	11.45
37 1/2	1470	1660	1243	8.91	1162	.0416	11.50	1.29069	19.53	19.53
38	1468	1668	1241	7.29	1420	.0416	11.50	1.57750	37.80	36.80
38 1/2	1463	1652	1235	5.76	1978	.0416	11.50	1.9965	92.20	92.20
39	1460	1649	1232	4.24	2440	.0416	11.50	2.71227	515.50	514.50
1/4	1458	1647	1230	3.52	2940	.02083	5.75	1.63353	43.01	42.01
1/2	1455	1643	1226	2.82	3670	.02083	5.75	2.0390	109.40	108.40
3/4	1452	1640	1223	2.16	4800	.02083	5.75	2.6620	459.40	458.4
40	1450	1638	1221	1.56	6640	.02083	5.75	3.6865	4858.	
1/4	1448	1634	1217	.90	11480	.02083	5.75	6.38889	2.4485	10^6
1/2	1445	1632	1215	.81	12880	.02083	5.75	7.09877	12.555	10^6
3/4	1443	1630	1213	.77	13440	.02083	5.75	7.4675	29.345	10^6
41	1440	1626	1209	.76	13620	.02083	5.75	7.5657	36.795	10^6

$$V^2 = \left(\frac{\frac{c}{W_x^2}V_1^2 - A}{\text{Antlg}\frac{276dx}{10W_x^2}} + A\right)\frac{1}{\frac{c}{W_x^2}} = \left(\frac{dV_1^2 - A}{b} + A\right)\frac{1}{d} = \frac{dV_1^2 + A(b-1)}{bd}$$

$$= \frac{dV_1^2 + Af}{bd}$$

C.R	bd	Af	d	dV_1^2	V^2	V
35					1.69	1.30
36	39,100	63,300	768	1296	1.65	1.28
36 1/2	7,740	10,070	859	1416	1.48	1.22
37	12,220	14,360	985	1461	1.29	1.14
37 1/2	22,720	23,070	1162	1502	1.08	1.04
38	53,700	45,700	1420	1531	.88	.94
38 1/2	181,400	114,000	1978	1737	.64	.8
39	1258,000	635,000	2440	1560	.507	.71
1/4	126,400	51,650	2940	1490	.42	.65
1/2	400,000	133,000	3670	1542	.336	.58
3/4	2200,000	550,000	4800	1610	.252	.503
40	32.2×10^6	$5.95\ 10^6$	6640	1672	.185	.43
1/4	28200×10^6	$2.98\ 10^9$	11480		.106	.326
1/2	16.16×10^{10}	$1.526\ 10^{10}$	12880		.095	.309
2/4	39.38×10^{10}	$3.56\ 10^{10}$	13440		.0904	.301
41	50.10×10^{10}	$4.35\ 10^6$	13620		.087	.297

C.R.	V	V^2	$W_x^2 10^6$	$\frac{cV^2}{W_x^2}$	P_a	P_a',	p	1845p	ΣR	ΣR lbs/ sq. in.	Ø
0	0	0		0	2570	2230	2230	410	830	630	1600
$\frac{1}{2}$	1.90	3.60	44.20	750	2550	2210	1460	270	690	530	840
1	2.54	6.42	44.20	1330	2530	2200	870	160	580	440	230
2	2.87	8.21	44.20	1700	2510	2180	480	90	510	390	90
3	2.92	8.50	44.20	1760	2480	2150	380	70	490	370	10
5	2.93	8.51	44.20	1730	2420	2100	370	70	490	370	10
7	2.89	8.35	44.20	1690	2360	2040	350	60	480	370	20
10	2.84	8.01	44.20	1630	2280	1920	290	50	470	360	60
15	2.73	7.09	44.20	1530	2160	1850	320	50	470	360	60
20	2.57	6.60	44.20	1430	2040	1750	320	50	470	360	60
23	2.42	5.83	40.51	1380	1980	1670	290	50	470	360	60
26	2.23	4.96	33.99	1340	1930	1640	300	50	470	360	60
29	2.00	4.00	28.30	1310	1875	1580	270	50	470	360	60
31	1.83	3.34	24.01	1270	1840	1560	290	50	470	360	60
33	1.62	2.62	19.01	1260	1800	1530	270	50	470	360	60
35	1.33	1.79	13.32	1350	1780	1500	300	50	470	360	60

C'Recoil	V el.	V^2	$W_x^2 \frac{1}{10^6}$	$\frac{cV^2}{W_x^2}$	P_a	P_a',	p	1845p	R	$\frac{R}{A}$	Ø
36	1.15	1.32	10.24	1180	1760	1480	300	57	474	361	60
37	.96	.92	7.29	1150	1750	1480	330	61	478	364	34
38	.72	.52	4.20	1130	1740	1470	340	63	480	365	25
39	.35	.12	1.00	1100	1720	1460	360	66	483	368	8
40	.29	.095	.72	1080	1710	1455	375	69	486	370	5
41	.25	.08	.70	1050	1700	1450	400	74	491	374	26

DESIGN LAYOUT OF COUNTER RECOIL.

Divide the total length of recoil into four parts:

L_o' = from 0 velocity of counter recoil to max. velocity (a)

$L_o'' = L_o - L_o'$ L_o = one-half of counter recoil distance (b)

L_b' = last half of counter recoil distance - 6 inches (c)

L_b'' = last 6 inches of counter recoil (d)

$$(P'_a - \frac{cV^2}{W_x^2})A - \Sigma R = MV \frac{dv}{dx}$$

(a) Assume a max. velocity of 2.95 ft/sec. When velocity is max. $V \frac{dv}{dx} = 0$

$$\therefore (P'_a - \frac{cV^2}{W_o^2}) A - \Sigma R = 0$$

$$P'_a - \frac{cV^2}{W_o^2} = \frac{R}{A} \qquad \frac{[K(A'_a - A'_b + a_b)\frac{A}{A_a}V]^2}{175 \times W_o^2}$$

$$2130 - \frac{cV^2}{Wx} = 370$$

$$\frac{(\frac{1}{9} \times 4.602 \times 1.314 \times 2.93)^2}{(5.303 \times 13.23 \times 41.95)^2} = W_o^2$$

W_o =.00669 sq.in. from drawings W_o = .00665

Deviation .45%

(b) Orifice is constant during this period

W_o = .00665

$(P'_a - \frac{cV^2}{W_x^2})A - \Sigma R = mV \frac{dv}{dx}$ equation of forces with one unknown velocity

Find V at "L_o"

(c) Calculate the unbalanced force at the beginning of the interval. This force is constant up to six inches from the bettery position.

$$(P'_a - \frac{cV^2}{W_x})A - \Sigma R = mV \frac{dv}{dx} = \emptyset = 79$$

$$mV\,dv = \emptyset\,dx \qquad \tfrac{1}{2}mV^2 \int_{V_1}^{V_x} = \emptyset x$$

$$\tfrac{1}{2}m(V_1^2 - V_x^2) = \emptyset_x$$

$$V_1^2 = \frac{2\emptyset x}{m} = V_x^2 = V_1^2 - \frac{2 \times 60}{32.6} x' = \frac{2 \times 60}{32.6 \times 12} x'$$

$$V_x = \sqrt{V_1^2 - .307 x''}$$

C'Re-coil	x"	.3x	V_1^2-.3x"	V_r	V_r	diff.	%
20.5	0	0	6.44	2.54	2.54	00	
23	2.5	.77	5.64	2.38	2.42	.04	1.65
26	5.5	1.67	4.77	2.19	2.23	.04	1.79
29	8.5	2.60	3.84	1.96	2.00	.04	2.00
31	10.5	3.22	3.22	1.80	1.83	.03	1.66
33	12.5	3.83	2.61	1.61	1.62	.01	.62
35	14.5	4.5	1.92	1.37	1.35	.02	1.48
36	15.5			1.14	1.15	.01	.87

(d) Velocity at the beginning of the period = V =1.14

$$\tfrac{1}{2}m_r(V_2^2-V_x^2) = \frac{\phi L_b''}{2}$$

$$\phi = \phi - \frac{\phi}{L_b''}x = \phi(1-\frac{x}{L_b''})$$

$$\phi=\phi(1-\frac{x}{L_b''}) = -mV\frac{dv}{dx}$$

$$\phi(1-\frac{x}{L_b''}) = -\tfrac{1}{2}m(V_1^2-V_x^2)$$

$$(\phi x-\frac{\phi x^2}{2L}) = \tfrac{1}{2}m(V_1^2-V_x^2)$$

$$V_x^2 = V_1^2 - \frac{2\phi x}{m}+\frac{\phi x^2}{Lm} = V_1^2+\frac{\phi}{m}(\frac{x^2}{L_b''}-2x) =$$

$$V_1^2-\frac{\phi}{m}(2x-\frac{x^2}{L_b''})$$

$$\phi = 82$$

$$L_b'' = 5"$$

$$V_x^2 = V_1^2-\frac{\phi}{m}(2x-\frac{x^2}{L_b''}) = V_1^2 - 2.51(2x-24x^2)$$

$= V_1^2 - 2.51(\frac{1}{6} x''- .0167 x^2) = V_1^2 - 2.51(.167-.0167)$

$= V_1^2 -.39$

C.R.	x	V_1^2	design V_1	drawing V	V_1-V_1	%Deviate
36	1	1.31	1.15	1.15	0	0
37	1	.92	.96	.96	0	0
38	1	.53	.73	.72	.01	1,37
39	1	.14	.37	.35	.02	5.40
40	1	0	0	.09	.09	
41	1	0	0	.08	.08	

Assuming orifice curve a parabola

$y^2=Kx$

$W_x^2=Kx$

$K = \frac{\overline{13.3}^2}{20} = 8.8445$

$W_x^2 = 8.8448x$

x	W_x^2	W_x	Orifice by parabola	Orifices drawing	a-b	%Deviation
1	8.84	2.98	.00147	.00125	.00022	16
2	17.69	4.20	.00210	.00206	.00004	2
4	35.38	5.95	.00297	.00320	.00023	7
6	53.07	7.30	.00365	.00410	.00045	11
8	70.76	8.41	.00420	.00465	.00045	10
10	88.45	9.45	.00470	.00513	.00043	8
12	106.13	10.30	.00515	.00550	.00045	8
14	123.82	11.11	.00555	.00583	.00028	5
16	141.51	11.89	.00594	.00612	.00018	3
18	159.20	12.61	.00630	.00640	.00010	1.5
20	176.89	13.30	.00665	.00665	.00000	0

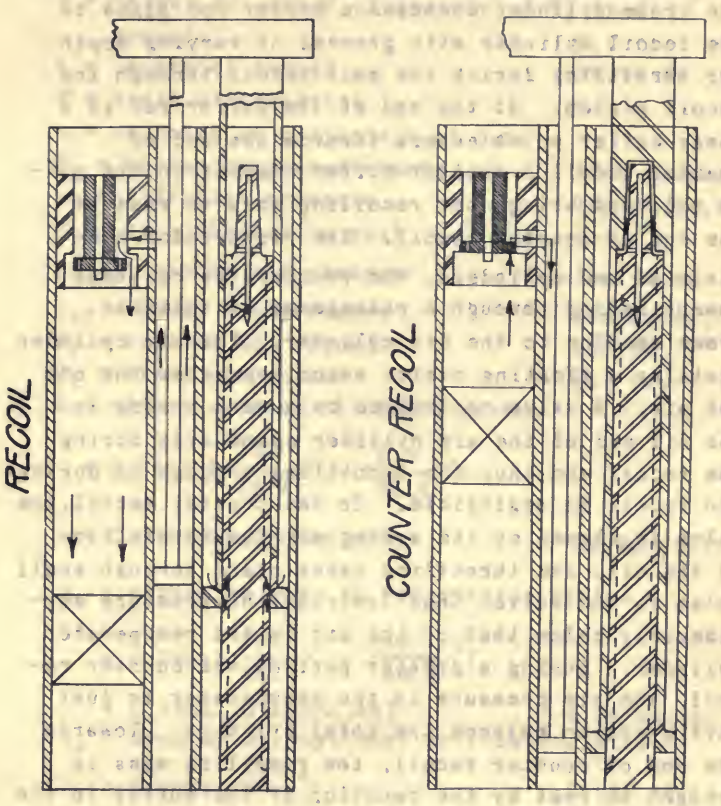


Fig. 10

RECOIL SYSTEM WITH VARIABLE RECOIL-INDEPENDENT RECUPERATOR.

In a mechanism of the type now to be discussed the brake cylinder contains a buffer rod fixed to the recoil cylinder with grooves of varying depth for throttling during the main recoil through the recoil piston. At the end of the buffer rod is a spear buffer which enters towards the end of counter recoil a smaller buffer chamber in the piston rod, and brings the recoiling mass to rest at the end of counter recoil. The recuperator consists of two cylinders, the recuperator cylinder communicating through a passageway of suitable cross section to the air cylinder. The air cylinder contains a floating piston which separates the oil and air. A valve controlled by a weak spring in the oil end of the air cylinder opens wide during the recoil and thus the throttling through it during the recoil is negligible. In the counter recoil, the valve is closed by its spring and the return flow of the oil, and throttling takes place through small holes in the valve, thus lowering the pressure considerably below that of the air in the recuperator cylinder. During a greater part of the counter recoil, the air pressure in the recuperator is just sufficient to balance the total friction. Towards the end of counter recoil, the recoiling mass is brought to rest by the reaction of the buffer in the hydraulic braking cylinder.

EXAMPLE AND CALCULATION. As an example of a successful model, a complete calculation will be given.

GENERAL DATA.

A_b' = area of bore	29.2 sq.in.
w = weight of projectile	96.1 lbs.
w_r = weight of recoiling parts	9124 lbs.
$\bar{w}$ = weight of powder charge	26 lbs.

u = travel of shot in bore — 185.68 (in.)

p_{bm} = max. powder pressure on breech — 31,500 lbs/sq.in.

h = height of center of gravity of W_r from ground — 5.7 ft.

b = length of recoil — 70.78 ft.

ϕ = angle of elevation — 0°

v = velocity of projectile in bore

S_v = volume of powder chamber — 1334 cu.in.

p_h = hydraulic pressure — lbs/sq.in.

P_h = total hydraulic pull — lbs.

p_{ai} = initial air pressure — lbs/sq.in.

p_{af} = final air pressure — lbs/sq.in.

V_i = initial air volume — cu. in.

V_f = final air volume — cu. in.

A = effective area of recoil piston

$$= 0.7854\left[\left(\frac{190}{25.4}\right)^2 - \left(\frac{110}{25.4}\right)^2\right]$$

$$= 0.7854(55.955-18.755) \quad 29.22 \text{ sq.in.}$$

A_r = effective area of recuperator piston

$$= 0.7854\left[\left(\frac{82}{25.4}\right)^2 - \left(\frac{40}{25.4}\right)^2\right]$$

$$= 0.7854(10.427-2.48) \quad 6.24 \text{ sq.in.}$$

A_a = area of air cylinder or floating piston

$$= 0.7854\left(\frac{137}{25.4}\right)^2 \quad 22.85 \text{ sq.in.}$$

Free velocity curve

By Le Duc's formula

Velocity of projectile in bore $V = \frac{au}{b+u}$ where

$u = \frac{u''}{12}$ = travel of projectile in bore - in ft.

$$a = 6823 \times \Delta^{\frac{1}{12}} \left(\frac{\bar{w}}{w}\right)^{\frac{1}{2}} \qquad \Delta = \frac{\bar{w} \times 27.68}{S_v}$$

$$= \frac{26 \times 27.68}{1334} = .5395$$

$$a = 6823 \times .5395^{\frac{1}{12}} \left(\frac{26}{91.6}\right)^{\frac{1}{2}}$$

$$\log a = \log 6823 + \tfrac{1}{12}(\log 5395 - \log 1000) + \tfrac{1}{2}(\log 26 - \log 96.1) = 3.83398 + \tfrac{1}{12}(3.73199 - 4.00000) + \tfrac{1}{2}(1.41497 - 1.98272) = 3.83398 + .31100 - .33333 + .707485 - .99136 = 3.52778$$

$$a = 3370$$

$$b = \frac{4W}{27} \times \frac{1.12a^2}{gA_gP_m} = \frac{4 \times 91.1 \times 1.12 \times 3371^2}{27 \times 32.2 \times 29.2247 \times 31500} = 6.1124$$

$$v_m = \frac{3371 \times 154733}{6.1124 + 15.4733} = \frac{52161}{21.5857} = 2416 \text{ ft/sec.}$$

V_f = free velocity of recoiling parts = $\dfrac{V_p(W + .5\bar{w})}{W_r}$

X_f = free displacement = $\dfrac{u(W+\bar{w})}{(W_r+W+\bar{w})}$ = .01288u = .01196 v

u	u in.	au	b+u	$V = \frac{au}{1+u}$	V_f .01196V	X_f .01288V
1	12	3370	7.1124	474	5.67	.155
2	24	6740	8.1224	832	9.95	.294
3	36	10110	9.1124	1110	13.28	.464
4	48	13460	10.1124	1330	15.91	.618
5	60	16820	11.1124	1512	17.99	.774
6	72	20200	12.1124	1667	19.84	.928
8	96	26900	14.1124	1900	12.61	1.235
10	120	33700	16.1124	2090	24.87	1.545
12	144	40400	18.1124	2230	26.54	1.854
13	156	43800	19.1124	2290	27.25	2.010
14	168	47200	20.1124	2340	27.85	2.161
15	180	50550	21.1124	2390	28.44	2.32
15.473	186	52150	21.5854	2410	28.69	2.392

CALCULATION OF E AND T.

$T = t_o + t_{i_o}$ where t_o = time, shot to muzzle

t_{i_o} = time, gas expansion

$$t_o = \frac{3}{2}\frac{u}{V_n} = \frac{3}{2} \times \frac{15.473}{2416} = .00963 \text{ seconds}$$

$$t_{i_o} = \frac{2(V_{fm} - V_o)W_r}{P_{ob}\ S_o\ g}$$ where V_{fm} = velocity of free recoil, mas.

V_o = velocity of free recoil when shot is leaving muzzle

W_r = weight of recoiling parts

P_{ob} = muzzle pressure at base of breech

S_o'' = cross section of the bore in sq. in.

$$V_{f_{max}} = \frac{W\ V_m + 4700\ \bar{w}}{W_r} = \frac{96.1 \times 2416 + 4700 \times 26}{9124} = 38.78 \text{ ft/sec.}$$

$$V_o = \frac{V_m(w + 0.5\bar{w})}{W_r} = \frac{2416(96.1 + 0.5 \times 26)}{9124} = 28.83 \text{ ft/sec.}$$

P_b = max. pressure on breech = $1.12\ P_m$ = 35,300 lbs/sq. in.

P_e = mean coast pressure on breech

$$= \frac{W\ V_m^2}{64.4\ nS_a''} = 19200 \text{ lbs/sq.in.}$$

$$e = u''\left\{\left(\frac{27}{16}\frac{P_m}{P_e} - 1\right) \pm \sqrt{\left(1 - \frac{27}{16}\frac{P_m}{P_e}\right)^2 - 1}\right\} = 57.38$$

$$P_{ob} = \frac{27}{4} e^2 \frac{u''}{(e+u'')^3} P_b \quad \frac{27}{4} \times \overline{57.38}^2 \times \frac{185.68 \times 35300}{(57375 + 185.68)^2}$$

$$= 10141 \text{ lbs/sq.in.}$$

$$t_{i_o} = \frac{2(V_{fm}-V_o)W_r}{P_{ob} \times A_a'' \times g} = \frac{2(38.78-28.83)9124}{10.141 \times 29.225 \times 32.2} = .01899$$

$T = t_o + t_{i_o} = .009631 + .01899 = .0286$ seconds.

$E = X_{fo} + X_{f'o} = S_f$ while shot travels to muzzle
$+ S_f$ gas cap.period.

$$X_{fo} = u\left(\frac{w+.05\bar{w}}{W_r+W+w}\right) = 15.473\left(\frac{96.1+13}{912+96.1+26}\right) = .1826 \text{ ft.}$$

$$X_{fo} = \frac{S_a'' P_{ob}}{W_r} g \frac{t_{E'o}^2}{3} + V_o t_{E'o} = \frac{32.2 \times 29.22 \times 10140 \times .01899^2}{3. \times 9124 \times}$$

$$+ 28.83 \times .01899$$

$= .5476$ ft.

$E = .1826 + .5476 = .7308$

$$t_o = \frac{b}{2}\left(2.3 \log \frac{2u}{b} + \frac{u}{b} + 2.098\right)$$

$$\frac{6.1124}{3370}\left(2.3 \log 0.327\, u + \frac{u}{6.1124} + 2.098\right)$$

$t_o = .00414 \log 0.327\, u + .000299\, n + .00378$
time up to (shot to muzzle)

$$t_{i_o} = \frac{2(V_f - V_o)}{P_{ob} \times S_a'' \times g} \quad \frac{W_r}{g} =$$ will give time corresponding to V_f during gas expansion period

$$t_o = \frac{0.00412}{B} \log \frac{0.0277\, u''}{A} + \frac{0.0000249\, u''}{C} + 0.00378$$

X_f	u"	0.0277u"	log A	B	C	t_o
.354	36	1.	0	0	.0009864	.00468
.566	48	1.3296	.12385	.000513	.0011952	.00549
.920	78	2.1606	.33465	.001385	.001942	.00710
1.345	114	3.1578	.49941	.002068	.002839	.00869
1.628	138	3.8226	.58240	.002411	.003436	.00963
1.840	156	4.3212	.63558	.002631	.003884	.01049
2.053	174	4.8198	.68305	.002828	.004333	.01094
2.191	185.65	5.1425	.71122	.002944	.004623	.01134

$$t_{1_o} = \frac{2W_r}{P_{ob} \times S_o'' \times g} (V_f - V_o) = .001912(V_f - V_o)$$

	V_f	V_o	$V_f - V_o$	t_{1_o}		t_o
3.00	31.30		2.47	.00472		.01606
4.00	33.66	28.33	4.83	.00924		.02058
5.00	35.40		6.57	.01256		.02390
6.00	36.85		8.02	.01534		.02668
7.00	37.81		8.98	.01717		.02851
8.00	38.55		9.72	.01859		.02993
8.76	38.68		9.85	.01883		.03017

$$\frac{K}{\text{Resistance to recoil.}}$$

Slope of the stability curve $= \frac{W_r b}{h} = R - r$ where

b = length of recoil

h = height of center of gravity of recoiling parts from the ground (5.70)

W_r = weight of the recoiling parts.

R = max. resistance to recoil in battery

r = max. resistance to recoil out of battery

$$R - r = \frac{W_r b}{h} = \frac{9124 \times 70.78}{68.4} = 9450$$

Total variable resistance to recoil = K =

$$K = \frac{\frac{W_r}{2g} V_{f_{max}}^2 + \frac{R-r}{2b}(b-E)^2}{b - E + V_f T - \frac{R-r}{b}(b-E)\frac{T^2}{2W_r}}$$

$$K = \frac{\frac{9124}{2 \times 32.2} \times 38.78^2 + 800(5.9058 - .730)^2}{5.9058 - .730 + 38.78 \times .0286 - 1600(5.9058 - .730)\frac{(.0286)^2}{2\frac{9124}{32.2}}}$$

$K = 37200$ lbs. in battery.

$$K' \text{ out battery} = K - :\left(\frac{R-r}{b}\right)\left(b-E+\frac{KT^2}{2M_r}\right) = 37200-1600(5.9058$$

$$-.730+.0538)$$

K' = 29600 lbs. out battery

Construct a curve showing K at each point of recoil.

CALCULATION OF AIR PRESSURE.

A = effective area of recuperator piston

V_i = initial air volume 1620 sq.in.

V_f = final air volume = $V_i - A_x$

P_{ai} = initial air pressure = 1420 lbs/sq.in.

$$P_{af} = \text{final air pressure} = P_{ai}\left(\frac{V_i}{V_f}\right)^{1.3}$$

$$= 1420\left(\frac{1620}{1620-6.25}\right)^{1.3}$$

$$\log P_{af} = \log 1420+1.3[\log 1620-\log(1620-6.25x)]$$

$$\log P_{af} = \log 1420+1.3[\log 1620-\log(1620-6.25\,x)]$$

$$P_{af} = 1420 \log 1.3\ e$$

Re-coil X	6.25 x	1620-6	log c	e	1.3e	c log 1.13e	P_{af}	c're-coil
1	6	1614	3.20790	.00162	.00211	1.0049	1430	70.78
5	31	1589	3.20112	.00840	.01092	1.0255	1460	65.78
10	63	1557	3.19229	.01723	.02240	1.0529	1500	60.78
15	93	1527	3.18384	.02568	.03338	1.0799	1530	55.78
20	124	1496	3.17493	.03459	.04497	1.1090	1580	60.78
25	155	1465	3.16584	.04368	.05678	1.1405	1620	45.78
30	186	1434	3.15655	.05297	.06886	1.1750	1670	40.78
35	217	1403	3.14706	.06246	.08120	1.2055	1710	35.78
40	248	1372	3.13735	.07217	.09382	1.2440	1770	30.78
45	279	1344	3.12743	.08209	.10672	1.2788	1820	25.78
50	313	1307	3.11628	.09324	.12121	1.3220	1880	20.78
55	341	1279	3.10687	.10265	.13345	1.3595	1930	15.78
60	375	1245	3.09517	.11435	.14866	1.4080	2000	10.78

	65	406	1214	3.08422	.12530	.16289	1.4550	2070	5.78
	68	425	1195	3.07737	.12215	.17180	1.4850	2110	2.78
	70	438	1182	3.07262	.13690	.17797	1.5068	2140	.78
70.	78	439	1181	3.07225	.13729	.17845	1.5080	2140	0

V_r

Velocity of retarded recoil.

During T

$V_r = V_f - \frac{K}{M_r} t$ where K = retarding or breaking force constant, during T. = 37200 lbs.

$V_r = V_f =$

$$\frac{37200 \times 32.2}{9124} t = V_f = 131.28\ t$$

$X_r = X_f = \frac{K}{2M_r} t^2$ where X_r = retarded displacement

$$X_r = X_f = \frac{37200 \times 32.2}{\times 9124} t^2 = X_f = 65.64\ t^2$$

t	t^2	X_f	$65.64t^2$	X_r	V_f	131.28f	V_r
.01085	.000122	2.19	.008	2.1	28.83	1.32	27.51
.0128	.000164	2.954	.01075	2.83	31.5	1.68	29.82
.0148	.00022	4.734	.01445	4.56	35.	1.94	33.06
.02984	.00089	8.76	.0585	8.06	38.78	3.91	34.87

V_r

After T,

$$K_x = K + \frac{R-r}{b}(b-x) = 29600 + 1600(5.9058 - x) = 39\,060 - 1600\ x$$

$$V_{r_2} - V_{r_1}^2 = -\frac{K+K_x}{m_r}\left[x - \left(E - \frac{KT^2}{2M_r}\right)\right] = V_{r_1}^2 - J$$

$$= V_r^2 - \frac{37200 + K_x(x - .737)}{283}$$

x"	x'	K_x	$\frac{K+K_x}{m_r}$	x-.73	J	V_x^2	V_r
9	.75	36860	265	.02	5.30	1215	34.90
19	1.585	36520	261	.855	223	997	31.6
29	2.418	35200	256	1.688	432	788	28.0
39	3.25	33860	251	2.52	633	587	24.2
49	4.08	32530	246	3.35	825	395	19.9
59	4.91	31210	242	4.18	1010	210	14.5
69	5.75	29860	237	5.02	1190	30	5.48
70.78	5.9	29610	236	5.17	1220	0	0

$$w = \frac{C\ A\ V_r}{13.22\ \sqrt{P_h}} \qquad C = 1.39 \qquad A = 29.22$$

X_r	V_r	P_h	$\sqrt{P_h}$	$13.22\sqrt{P_h}$	CAV_r	w
2.1	27.51	955	30.9	408	1110	2.72
2.825	29.82	950	30.8	407	1200	2.95
4.561	33.06	945	30.78	406	1330	3.28
8.06	34.87	940	30.62	405	1405	3.47
9.	34.90	936	30.60	404	1410	3.49
19.	31.60	885	29.78	393	1275	3.24
29.	28.	815	28.50	377	1130	3.
39.	24.2	738	27.2	360	975	2.71
49.	19.9	662	25.8	340	803	2.36
59.	14.5	577	24	318	585	1.74
69.	5.4	492	22.2	394	221	.752
70.87	0	477	21.82	289	0	0

As the total oil displaced by the recoil piston does not pass through the recoil throttling grooves, a part passing into the buffer chamber the above throttling formula, is not exact and becomes

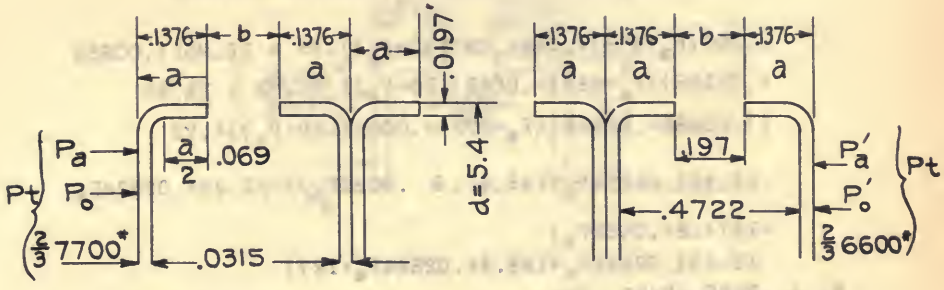
$$w = \frac{C(AV_r - Qb)}{13.22\sqrt{P_h}}$$

where Qb= the quantity of oil passing into the buffer chamber.

With this value of w. C should be given a value of about 1.5.

FRICTION CALCULATION.

Floating piston friction = R_f



$R_f = K\ A\ P\ f$

P_a = air pressure (lbs/sq.in.)

P_o = belv. spring pressure 42F= load 7700 lbs.

$$\frac{7700}{.7854(\overline{5.38}^2 - \overline{4.37}^2)} = 1000 \text{ lbs/sq.in.}$$

P'_o = belv. spring load 42E= 6600 lbs.= 860 lbs/sq.in.

K = poisson ratio

f = coefficient of friction

A = area of frictional surface

P_t = intensity of pressure $\perp$ to A

$$R_f = \pi d[(bf+af')P_t + \tfrac{1}{2}\, af'P_g]K + \pi d[(bf+af')P'_t + \tfrac{1}{2}af'P_g]K$$

d = 5.4 in.

b = .197 in.

a = .1376 in.

K = poisson ratio = .73

P_t = air pressure + blv. spring pressure (P_a+P_o)

$= P_a + \frac{2}{3}$ 1000 lbs/sq.in.

P'_t = oil pressure + blv. spring pressure (P'_a+P_o)

$= P_a + \frac{2}{3}$ 860 lbs/sq.in.

$$P_g = \text{spiral spring pressure} + P_a \frac{3000}{.7854\times 5.4^2} + P_a$$

$= 130 + P_a$ lbs/sq.in.

Assume $P_a = P'_a$

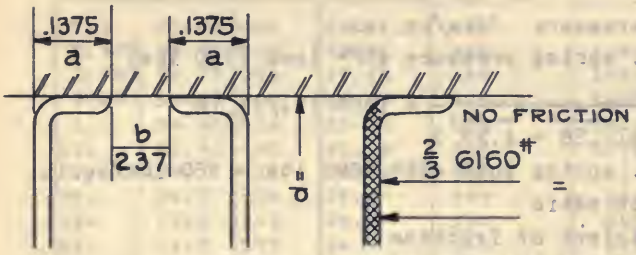
$R_f = 3.1416 \times 5.4[(.197 \times .05 + .1376 \times .09)(P_a + 668) + .069 \times .09(130 + P_a)].73 + 3.1416 \times 5.4[(.197 \times .05 \times .1376 \times .09)(P_a + 573) + .069 \times .09(130 + P_a)].73 = 16.95[(.00986 + .01238)(P_a + 668) + .0062(130 + P_a)] \times .73 + 16.95[(.00986 + .01238)(P_a + 573) + .0062(130 + P_a)] \times .73$

$12.38(.02224P_a + 148.4 + .8 \quad .0062P_a) + 12.38(.02224P_a + 127 + .8 + .0062P_a)$

$12.38(.02844P_a + 148.4 + .02844P_a + 127)$

$R_f = .352P_a + 3430$ lbs.

RECUPERATOR PISTON HEAD FRICTION = R_2

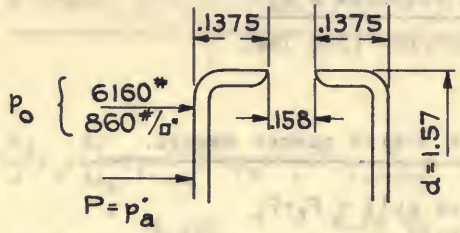


$R_2 = K\ A\ P\ f = \pi d(af' + bf)(P + P_o)K = \pi \times 3.225(.1375 \times .09 + .237 \times .05)(980 + P).73 = 10.14 \times .0242(980 + p).73$

$R_2 = 0.18\ P_o + 175$ lbs.

RECUPERATOR PISTON ROD PACKING FRICTION

$= R_3$

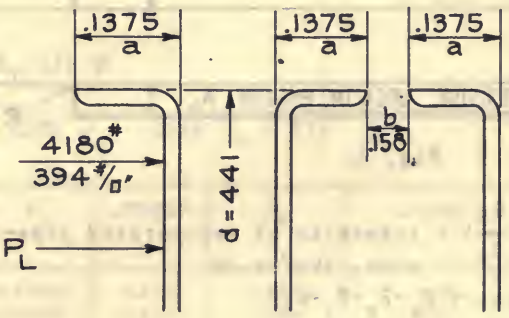


$R_3 = \pi \times 1.57(.1375\times.09+.158\times.05)(860+p\ 6\ ).$

$= 4.94\times.0203(860 + p).73 = 63+.073\ p$

$R_3 = 60+.073\ p_a$ lbs.

RECOIL BRAKE STUFFING BOX FRICTION $= R_4$



$A_{3\,5\,D} = .7854(32.85-19.4) = 10.6$

$$\frac{4150}{10.6}$$

$R_4 = \pi \times 4.41(\frac{3}{2}\ .1375\times.09+.158\times.05)(394+P_g).73$

$= 10.11(.0186 + .0079)(394 + p_g)$

$R_4 = 105 + .268\ p_h$ lbs.

GUIDE FRICTION = R_g

$R_g = .15W_r = 9124 \times .15 = 1370$ lbs.

$R_g = 1370$

FORCES ACTING DURING RECOIL.

B = total braking pull = $P_h + P_a$

P_h = hydraulic pull

P_a = load on air recuperator pull

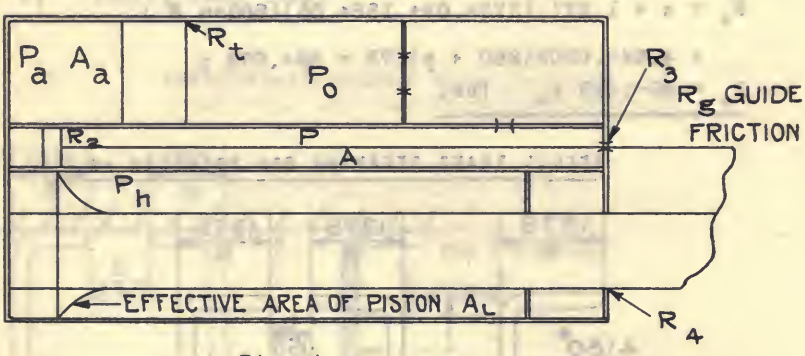


Fig. A.

$p_a' = (P_a + \frac{R_f}{A_a})$ = intensity of recuperator pressure, lbs/sq.in.

R_t = friction = $R_2 + R_3 + R_4 + R_g$

$B + R = P_h A + p_a' A_a + R_t$

$R_2 = 0.18\ P_a + 175$
$R_3 = .073P_a + 60$
$R_4 = .268P_h + 105$
$R_g = 1370$
} lbs.

$\Sigma R = R_t = .253P_a + .268P_h = 1710$

$$P = K + W_r \sin \phi$$

$$P_h = \frac{P-(p_a' A_v + R_t)}{A} = \frac{p-[(p_a+\frac{R_f}{A_a})A_v + 2.253p_a + 0.268p_h + 1710]}{A}$$

$$= \frac{p-(p_a A_v + \frac{A_v}{A_a}R_f + 0.253p_a + 0.268p_h + 1710)}{A} = \frac{p}{A+.268}$$

$$-(\frac{A_v}{A}p_a + \frac{A_v}{AA_a}R_f + \frac{0.253}{A}p_a + \frac{1710}{A})\frac{A}{A+.268}$$

$$= \frac{p}{26.268} - \frac{6.493}{26.268}\ p_a - \frac{6.24\ R_f}{22.85 \times 26.268} - \frac{1710}{26.268}$$

$$= .0381\ p - 0.2472\ p_a - .2376\frac{R_f}{22.9} - 65$$

but $R_t = .352\ p_a + 3430$

$$\frac{R_f}{A_a} = .0154\ p_a + 150 \quad \text{hence } p_h = .0381\ p - 0.2626\ p_a - 215 \text{ (lbs/sq.in)}$$

$$P = K + W_r \sin \phi$$

$$P_h = .0381\ P - .2626\ P_a - 215$$

Recoil	P	.0381P	P_a	.2626P_a	P_h
1	37200	1417	1430	376	826
5	37200	1417	1460	383	819
10	37000	1410	1500	394	801
15	36400	1387	1530	401	771
20	35800	1364	1580	415	734
25	35200	1341	1620	425	701
30	34600	1318	1670	439	664
35	34000	1295	1710	450	630
40	33500	1276	1770	465	596
45	32900	1253	1820	478	560
50	32300	1231	1880	494	522
55	31700	1208	1930	506	487
60	31200	1189	2000	525	449

65	30600	1166	2070	544	407
68	30200	1152	2110	555	382
70	30000	1143	2140	563	366
70.78	29600	1128	2140	563	352

$\Sigma R = R_f + R_2 + R_3 + R_4 + R_g$

$R_f = (.352 p_a + 3430) \frac{6.25}{22}$

$(.01 p_a + 975)$

$R_2 = .180\ p_a + 175$

$R_3 = .073\ p_a + 60$

$R_4 = 105 + .268\ p\ h$

$R_g = 1370$

$\Sigma R = .263\ p_a + .268\ p\ h + 2685$

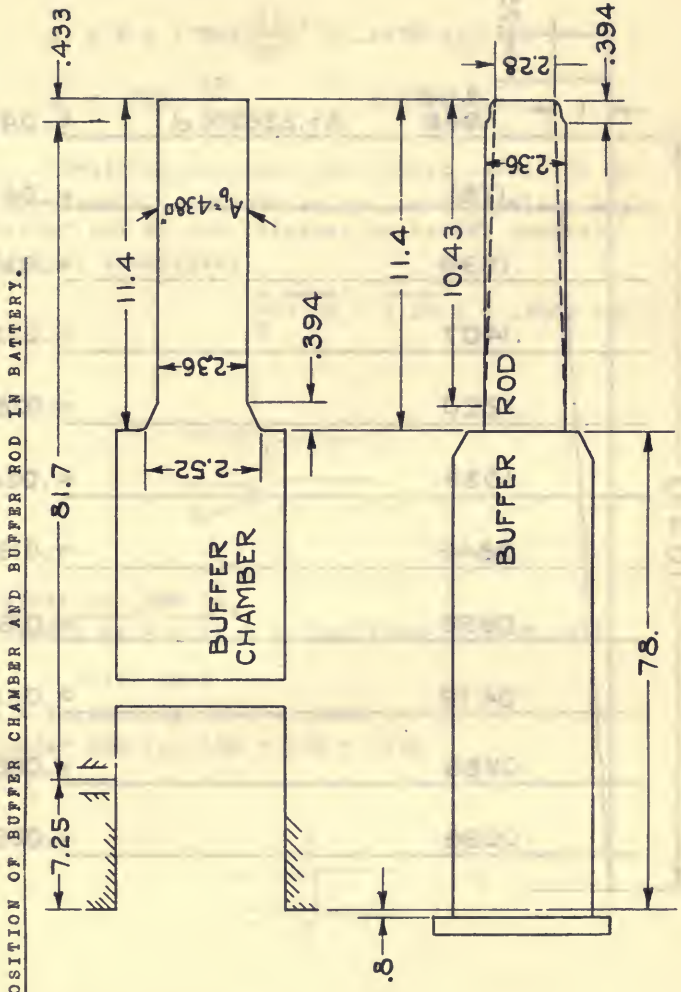
Recoil	$.263 p_a$	$.268 ph$	R	phA	Ap_a
5	380	540	3605	21250	9130
15	400	540	3625	20000	9550
25	425	530	3640	18200	10110
35	450	530	3665	16350	10700
45	478	520	3683	14550	13800
55	510	520	3715	12650	12050
65	545	520	3750	10600	12920
70	560	510	3755	9530	13380
70.87	560	510	3750	9160	13380

COUNTER RECOIL CALCULATIONS.

Oil Ports

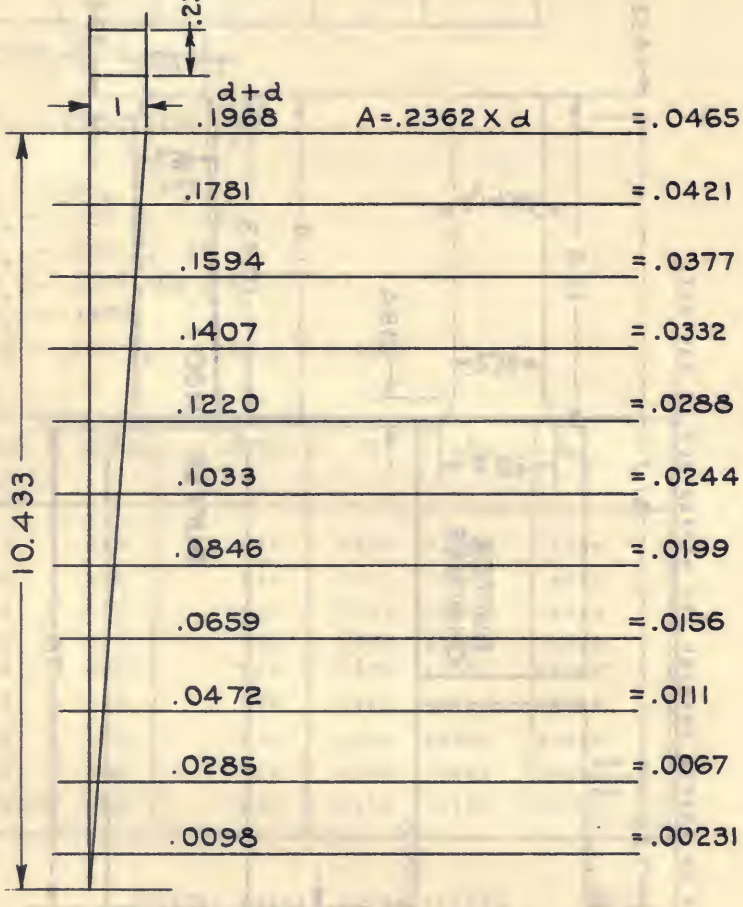
Recuperator valve oil ports: see fig. (A) Page 59.
Ports in 58B1

$W_o = 2 \times .7854 \times (\frac{4}{25.4})^2 = .03896$ sq.in.



POSITION OF BUFFER CHAMBER AND BUFFER ROD IN BATTERY.

BUFFER GROOVE.



Recoil brake :- filling in buffer port holes
Port in piece 29A

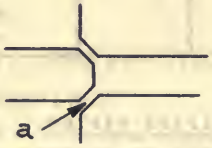
$a_o = a_o'$ diameters 15.mm.

$$a_o = 2 \times .7854\left(\frac{15}{24}\right)^2 = .5478 \text{ sq.in.}$$

$$a_o = .7854\ \frac{20}{24} \qquad .4869 \text{ sq.in.}$$

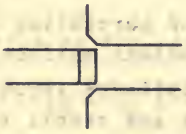
Smallest oil port in counter recoil is W_o
therefore W_o = .03896 sq.in. is throttling area.
Buffer rod at the entrance to buffer chamber.
Area of throttling

$$\frac{\pi}{4}(\overline{2.52}^2 - \overline{2.28}^2) = .9048 \text{ sq.in.}$$



Buffer $\underline{\text{in}}_2$ $\underline{394\ \text{in}}_2$
$.7854(2.36 - 2.28) + 2362[1968-(2.36-2.28)]$

.3712+.0276
a = throttling area = .3988
Buffer rod in .394 +.394 = .788



throttling area = .045

THROTTLING AREA

Clearance for the buffer rod $\pi[(\frac{59.99}{25.4})^2 - (\frac{59.93}{25.4})^2]$

$= .0044$

	W_x	Clear a	
0	$W_o = .0389$		
62.11	.0358		.0389
62.78	.0315		.0359
63.78	.0270		.0314
64.78	.0227	.0044	.0281
65.78	.0181		.0285
66.78	.0135		.0179
67.78	.0090		.0134
68.78	.0047		.0091
69.78	0		.0044
70.78	0		.0044

CONSTANT ORIFICE PERIOD.

ACCELERATING PERIOD OF COUNTER RECOIL WITH CONSTANT ORIFICE.

Let p_a = air pressure (lbs/sq.in)
R_f = floating piston friction (lbs)
A_a = cross section area of floating piston (sq.in)
A_v = effective area of recuperator piston (sq. in)
K = reciprocal of throttling constant
w_o = area of constant orifice in valve of recuperator cylinder (sq.in)
ΣR = total guide and recoil friction (lbs)

Now $[(p_a - \frac{R_f}{A_a}) - \frac{K^2 A_v^2 v^2}{175 w_o^2}] A_v - \Sigma R = m_r v \frac{dv}{dx}$ since the throttling through the various orifices in the brake cylinder was

found to produce a negligible braking reaction as compared with the equivalent throttling in the recuperator system. Hence

$$(p_a - \frac{R_f}{A_a})A_v - \frac{K^2 A_v^3 v^2}{175\, w_o^2} - \Sigma R = m_r v \frac{dv}{dx}$$

Assuming a uniform velocity attained at 3 ft/sec., we have constant throttling area,

$$\frac{K^2 A_v^3 v^2}{175\, w_o^2} = (P_a - \frac{R_f}{A_a})A - \Sigma R$$

$$W_o = \sqrt{\frac{K^2 A^3 V^2}{175[(P_a - \frac{R_f}{A_a})A - \Sigma R]}}$$

If $K = \frac{1}{g} = 1.11$

$R_2 = .253\, P_a + .268\, ph + 1710$

$360 + 220 + 1710$

$\Sigma R = 2300$ lbs.

$$W_o^2 = \frac{\overline{1.11}^2 \times \overline{6.24}^3 \times \overline{3}^{-2}}{175[(1420 - 172)\, 6.25 - 2300]} = \frac{18.452}{737}$$

$$= .1543 \times \overline{10}^2$$

$$W_o = .0398 \text{ sq. in.}$$

Constructively we have $W_o = .03896$ sq.in.

COMPUTATION OF THE COUNTER RECOIL:

$$(P_a - \frac{R_f}{A_a})A_v - \frac{KV^2}{w_o^2} - \Sigma R = m_v\, v \frac{dv}{dx}$$

$$P_a A - \frac{R_f}{A_a} A_v - \frac{KV^2}{w_o^2} - (.253 p_a + 100 + 1710) = m_v\, V \frac{dv}{dx}$$

$$P_a A_v = \frac{.352\ p_a + 3430}{22.85} A_v - .253\ p_a - 1810 = m_v V \frac{dv}{dx}$$

$$6.25\ p_a - .0963\ p_a - 938 - \frac{KV^2}{w_o^2} - .253\ p_a - 1810 = m_v\ V \frac{dv}{dx}$$

$$5.9\ p_a - \frac{KV^2}{w_o^2} - 2750 = m_v\ V \frac{dv}{dx}$$

$$5.9\ p_a\ dx \frac{KV^2}{w_o^2} dx - 2750\ dx = \tfrac{1}{2} m (V_2^2\ V_1^2)$$

$$\frac{2 \times 5.9 P_a dx}{284} - \frac{2}{284} \frac{KV^2}{w_o^2} dx - \frac{2 \times 2750\ dx}{284} = V_2^2 - V_1^2$$

$$.0416\ P_a\ dx - \frac{K}{m w_o^2}\ V_1^2 + V_1^2\ dx - 19.35\ dx = V_2^2 - V_1^2$$

$$.0416 P_a\ dx - \frac{.6064\ dx\ V_2^2}{10^6 w_o^2} - \frac{.6064\ dx\ V_1^2}{10^6 w_o^2} - 19.35\ dx = (V_2^2 - V_1^2)$$

$$V_2^2 (1 + \frac{6064\ dx}{10^6 w_o^2}) = V_1^2 (1 - \frac{6064\ dx}{10 w_o^2}) + .0416 P_a dx - 19.35\ dx$$

$$w_o^2 = \frac{.0389}{2} = .001513$$

$$V_2^2 (1 + .334\ dx") = V_1^2 (1 - .334\ dx") + .00347\ p_a dx"$$
$$- 1.611\ dx"$$

C'Re-	dx"	p_a	.334dx	.00347p_a	.00347$p_a$dx	1.611dx	v
.25	.25	2.160	.0835	7.5	1.87	.402	1.165
.50	.25	2.150	.0835	7.45	1.865	.402	1.58
1.	.50	2.145	.167	7.43	3.72	.805	2.07
2.	1.	2.125	.334	7.37	7.37	1.611	2.54
4.	2.	2.100	.67	7.27	14.27	3.222	2.84
6.	2.	2.060	.67	7.15	14.30	3.222	2.87
10.	4.	2.000	1.33	6.94	27.76	6.45	2.82
15.	5.	1.950	1.67	6.76	33.80	8.05	2.77
20.	5.	1.900	1.67	6.6	33.	8.05	2.72
25.	5.	1.845	1.67	6.4	32.	8.05	2.66
30.	5.	1.800	1.67	6.25	31.25	8.05	2.63
35.	5.	1.730	1.67	6.	30.	8.05	2.55
40.	5.	1.700	1.67	5.9	29.5	8.05	2.53
44.	4.	1.670	1.33	5.72	23.19	6.45	2.51
48.	4.	1.600	1.33	5.55	22.8	6.45	2.48
51.	3.	1.600	1.	5.47	16.64	4.83	2.43

$$V_2^2\frac{(1+.334dx")}{a} = \frac{V_1^2\frac{(1-.334\,dx")}{b} + \frac{d}{.00347p_a dx"-1.611dx"}}{\Sigma}$$

C'Re-coil	dx"	a	b	P_a	c	d	Σ	V^2	v
51	3	200	0	1600	16.64	11.80	11.80	5.90	2.43
54	3	200	0	1580	16.45	11.61	11.60	5.80	2.41
56	2	1.668	.332	1560	10.81	7.59	9.52	5.71	2.39
58	2	1.668	.332	1540	10.67	7.45	9.35	5.60	2.365
59	1	1.334	.666	1530	5.32	3.71	7.44	5.57	2.36

BUFFER IN ACTION - RETARDED COUNTER RECOIL.

The dynamic equation during the buffer action period, becomes

$$(p_a - \frac{R_f}{A_a} - \frac{K^2A_v^2\,v^2}{175w_o^2})A_v - \frac{K^2A_b^2v^2}{w_x^2} - \Sigma R = m_r\,v\,\frac{dv}{dx}$$

Simplifying, we have $(p_a - \frac{R_f}{A_a})A_v - \frac{Kv^2}{w_o^2} - \frac{K_1v^2}{w_x^2} - \Sigma R = m_r\,v\,\frac{dv}{dx}$

Since $\Sigma R = 253\ p_a + 100 + 1710$ we have

$$p_a A_v - \frac{.352 p_a + 3430}{22.85} A_v - \frac{KV^2}{w_o^2} - \frac{K_1 V^2}{w_x^2} - 253\ p_a - 1810 =$$

$$m_r\ V \frac{dv}{dx}$$

$$5.9\ p_a - \frac{KV^2}{w_o^2} - \frac{K_1 V^2}{w_x^2} = 2750 = m_r\ V \frac{dv}{dx}$$

$$\frac{2 \times 5.9 p_a}{284} dx - \frac{2}{m} \frac{KV^2}{w_o^2} dx - \frac{2}{m} \frac{K_1 V^2}{w_x^2} dx \frac{2 \times 2750}{284} dx = V_2^2 - V_1^2$$

$$.041 p_a dx - m \times \frac{K}{w_o^2} V_2^2 + V_1^2\ dx - \frac{K_1}{m w_x^2} V_2^2 + V_1^2\ dx - \frac{2 \times 2750}{284}$$

$$dx - V_2^2 - V_1^2$$

$$.041\ p_a\ dx - \frac{6064}{10^6 W_o^2} dx V_2^2 - \frac{6064}{10^6 W_o^2} dx\ V_1^2 - \frac{K_1 (V_2^2 + V_1^2)}{m w_x^2} dx$$

$$= 19.35\ dx - V_2^2 - V_1^2$$

$$K_1 = \frac{R^2 A_b^3}{175} = \frac{1.5^2 \times 4.38^3}{175} = 1.0804$$

$$\frac{K_1}{m} = \frac{3804}{106}$$

$$.0416 P_a dx - \frac{6064}{10^6 w_o^2} V_2^2 dx - \frac{6064}{10^6 w_o^2} dx - \frac{3804 V_2^2}{10^6 w_x^2} dx -$$

$$\frac{3804 V_1^2}{10^6 w_x^2} dx - 19.35\ dx = V_2^2 - V_1^2$$

$$V_2^2 \left(1 + \frac{6064}{10^6 w_o^2} + \frac{3804}{10^6 w_x^2} dx\right) = V_1^2 \left(1 - \frac{6064}{10^6 w_o^2} dx - \frac{3804}{10^6 w_x^2} dx\right)$$

$$+ .0416\ P_a\ dx - 19.35\ dx$$

$$V_2^2\Big[1+\overbrace{\Big(334+\frac{317}{10^6 w_x^2}\Big)}^{b}dx''\Big]=V_1^2\Big[1-dx''\overbrace{\Big(.334+\frac{317}{10^6 w_x^2}\Big)}^{c}\Big]+\overbrace{(.00347P_a-1.611)}^{F}dx''$$

$$\underbrace{\qquad\qquad}_{a}\qquad\underbrace{\qquad\qquad}_{a}\qquad\underbrace{\qquad\qquad}_{d}$$

C'Re-coil	dx"	W_x	$\frac{317}{10^6 w_x^2}$	a	b	c	p_a	$.00347p_a$	d	f	cV_1^2+f	v^2
60	1	.0485	.127	.461	1.461	.539	1504	5.220	3.609	3.609	6.629	4.53
61	1	.0440	.164	.498	1.249	.751	1494	5.180	3.569	1.785	4.915	3.94
62	1	.0395	.203	.537	1.269	.731	1486	5.155	3.544	1.772	4.502	3.55
63	1	.0350	.259	.593	1.593	.407	1477	5.120	3.509	3.509	4.955	3.11
64	1	.0305	.341	.675	1.675	.325	1468	5.083	5.474	3.474	4.484	2.69
65	1	.0261	.466	.800	1.800	.200	1460	5.065	3.454	3.454	3.989	2.22
66	1	.0217	.674	1.008	2.008	.008	1452	5.040	3.429	3.429	3.411	1.70
67	$\frac{1}{2}$	.0174	1.045	1.379	1.689	.311	1444	5.015	3.404	1.702	2.176	1.29
68	$\frac{1}{2}$	.0130	1.875	2.109	2.065	.065	1435	4.980	3.379	1.690	1.635	.798
69	$\frac{1}{4}$	.0082	4.725	5.059	2.265	.265	1426	4.950	3.339	.835	.717	.317
$\frac{3}{4}$	$\frac{1}{4}$	.0043	17.590	17.924	5.481	3.481	1420	4.925	3.314	.829	.364	.066
70	$\frac{1}{4}$	.0043	17.590	17.924	5.481	3.481	1418	4.920	3.309	.827		

$$\frac{K A^2V^2}{175w_o^2} = \frac{\overline{1.11}^2 \times \overline{6.24}^2}{175\times \overline{0389}^2}V^2 = \frac{1.23\times 38.94}{2648}V^2 = \frac{48.07}{2648}V^2$$

$$= 181.53 = \frac{K^2A_b^2V^2}{175w_x^2} = \frac{\overline{1.5}^2 \times \overline{4.38}^3}{175\times 6.24 w_x^2}V^2 = .1731$$

$$\Sigma R = \frac{.352p_a+3430}{22.85} + \frac{.253p_a+1810}{6.24} = .0154p_a+150.11+.0405p_a + 280 = .056p_a+440$$

$$\frac{.352p_a+3430}{22.85} + \frac{.253p_a+.268+1710}{6.24}$$

$.0154p_a+150 + 043\ p_h$

$0483p_a+ 275$

$.0637p_a+.043p_h+430$

C'Re-coil	P_a	P_h	0637p	$.043p_h$	
1	2150	830	138	36	604
2	2130	825	136	36	604
4	2100	820	135	35	600
6	2060	815	132	35	500
8	2030	810	130	35	500
12	1980	800	127	34	500
20	1900	730	122	31	583
30	1800	660	116	28	574
40	1700	600	109	26	565
50	1580	520	101	22	553
60	1510	450	97	20	547
65	1480	410	95	17	542
68	1450	380	93	16	539
70	1440	370	92	16	538

COMPONENT FORCES.

C'Re-coil	P_a	V^2	KV^2	P_h-KV^2	$W_x^2 10^6$	K_1V^2	
.5	2150	2.50	4.20	1700			
1.	2150	4.28	790	1360			
2.	2125	6.44	1180	950			
4.	2100	8.15	1500	600			
6.	2060	8.24	1510	550			
10.	2000	8.01	1470	520			
15	1950	7.70	1420	530			
20	1900	7.45	1370	530			
25	1845	7.20	1320	530			
30	1800	6.90	1270	530			
35	1730	6.68	1220	510			
40	1700	6.43	1180	520			
45		6.20					
50	1580	6.00	1040	480			
55		5.74					
60	1500	4.53	830	670	2350	320	350
62	1490	3.72	690	800	1560	420	380
64	1460	2.86	530	930	930	540	390
66	1450	1.82	340	1110	484	660	450
68	1440	.87	160	1280	175	870	410
69	1430	.42	80	1350	72	1010	340

CONSIDERATION OF VARIOUS ELEMENTS OF DESIGN - HYDRAULIC BRAKE LAYOUT.

In the type discussed, the throttling is effected through grooves in the counter recoil buffer rod, there being two sets of grooves, one for horizontal elevation and long recoil and the other for max. elevation and short recoil.

The recoil throttling is not effected materially by the filling in buffer since the filling in process is not continuous throughout the recoil. Hence the throttling grooves are computed in the ordinary manner. The maximum throttling groove occurs at horizontal recoil near the beginning of recoil.

If $p_{h\ max}$ = the maximum allowable hydraulic pressure (assume 4000 lbs/sq.in)

C = the reciprocal of the contraction factor of the throttling orifice (assume C = 1.39)

V_r = the max. restrained velocity of recoil (assume $V_r = 0.9\ V_f$)(ft/sec)

$V_f = \frac{wv+4700\bar{w}}{w_r}$ = max. free velocity of recoil (ft/sec)

where w = weight of projectile (lbs)

$\bar{w}$ = weight of powder charge (lbs)

w_r = weight of recoiling parts (lbs)

v = muzzle velocity of projectile (ft/sec)

K_h = total resistance to recoil at horizontal or min. elevation (resistance to recoil should follow stability slope) (lbs)

K_s = total resistance to recoil at max. elevation (lbs) (made constant throughout the recoil)

P_h = total hydraulic pull (lbs)

F_{vi} = initial recuperator reaction (lbs)

F_{vf} = final recuperator reaction (lbs)

$m = \frac{F_{vf}}{F_{vi}}$ = ratio of compression (1.5)

n = coefficient of friction (0.2 to 0.3)

Then $F_{vi} = 1.3W_r(\sin \phi + n \cos \phi)$ the max. hydraulic pull at short recoil, max. elevation,

$P_{hm} = K_s+W_r(\sin \phi_m-0.3 \cos \phi_m)-F_{vi}$ (lbs) the max. hydraulic pull at long recoil, horizontal elevation.,

$P_{ho} = K_h -0.3W_r-F_{vi}$ (lbs)

The required effective area of the recoil piston then becomes,

$$A = \frac{P_{hm}}{P_{h\ max}} \text{ sq.in.}$$

Hence the max. throttling orifice, becomes,

$$W_{h\ max} = \frac{C\ A\ V_r}{13.2\sqrt{P_{ho}}} \quad \text{(sq.in)}$$

and the minimum throttling orifice, becomes,

$$W_{h\ min} = \frac{C\ A\ V_r}{13.2\sqrt{P_{h\ max}}} (sq.in)$$

The throttling areas decrease approximately in the recoil as a parabolic curve. For a satisfactory layout, for the cross section of the brake cylinder the inside cross section of the hollow recoil rod should be made approx. three times the maximum throttling orifice, that is three times the max. area of the four grooves in the counter recoil rod.

Hence tne inside diam. of the hollow piston rod, becomes,

$$d_i = \sqrt{\frac{3\ w_{h\ max}}{0.7854}} \text{ in.}$$

The diam. of the spear buffer chamber in the hollow recoil piston rod, is obtained by a consideration of the counter recoil stability and the maximum allowable buffer pressure.

The maximum buffer reaction on counter recoil becomes,

$$B' = F_{vi} + C_s' \frac{W_s l_s'}{h} - 0.3\ W_r$$

where F_{vi} = the initial recuperator reaction (lbs)

C_s' = constant of counter recoil stability

= 0.85

W_s = weight of entire system (lbs)

W_r = weight of recoiling parts (lbs)

l_s' = horizontal distance from W_s to wheel and ground contact (in)

The max. allowable buffer pressure, p_{bm}' = 1500 to 2500 lbs/sq.in. Hence the area of the spear buffer chamber in the recoil rod, becomes,

$$A_b = \frac{B'}{p_{bm}'} (sq.in)$$

When counter recoil stability is not of consideration it is customary to make the buffer stroke

about 17.5% of the length of horizontal recoil. Assuming an entrance velocity into the buffer at 2.5 ft/sec., we have,

$$B' = F_{vi} + \frac{\frac{1}{2} m_r v^2}{0.175\ b_h} \quad \text{where } v = 2.5 \text{ ft/sec.}$$

$$= F_{vi}\ 0.357\ W_r \quad \text{where } W_r = \text{weight of the recoiling parts, and the area}$$

of the spear buffer becomes,

$$A_b = \frac{B'}{P'_{bm}}$$

A further consideration requires the diam. of the buffer to be approximately a given ratio of the inside diam. of the recoil rod, in order that a proper layout for the filling in holes to the buffer may be made. Then,

$$\frac{d_{ib}}{d_i} = \frac{2}{3} \quad \text{or} \quad d_{ib} = \frac{2}{3} d_i$$

where $0.7854\ d_{ib}^2 = A_b$

The proper thickness of the buffer rod is based on a consideration of both the maximum longitudinal tension during the acceleration or powder period, and the compression stress due to an external pressure on a hollow cylinder, the internal pressure being zero. The max. allowable fibre stress in the brake rod will be taken,

$f_m = \frac{3}{8}$ to $\frac{1}{2}$ elastic limit (lbs/sq.in)

The outside diam. of the rod based on max. tension becomes

$$d_o^2 - d_i^2 \sqrt{\frac{P + \frac{w_p}{w_r}(P_b - B)}{0.7854\ f_m}}$$

where d_o = outside diam. of rod

d_i = inside diam. of rod

w_p = weight of piston + that portion of the rod (estimated) to the farthest hollow section from the piston.

w_r = weight of recoiling parts

P_b = max. powder reaction on breech

P_h = max. hydraulic braking.

To allow for the acceleration load $\frac{w_p}{w_r}(P_b - B)$ we may increase P_h to 1.3 P_h, then

$$d_o \sqrt{d_i^2 + 1.6 \frac{P_h}{f_m}}$$

which is sufficiently close in a preliminary layout. If we now consider the compression due to the hollow piston rod with an external compressive pressure, we have, $f_m(d_o - d_i) = 1.3\ p_{hm} d_o$

hence $d_o = \frac{d_i}{(1 - \frac{1.3 p_{hm}}{f_m})}$ approx.

where p_{bm} = max. pressure in the brake cylinder (lbs/sq.in)

The diam. of the recoil brake cylinder may now be calculated. We have $0.785\ d_r^2 = A + 0.785\ d_o^2$

where $A = \frac{K_s + W_r(\sin \phi_m - 0.3 \cos \phi_m') - F_{vi}}{P_h \text{ max.}}$

K_s = total resistance to recoil at max. elevation.

$P_{h\ max}$ = max. allowable pressure in recoil cylinder consistent with packing

= 4000 lbs/sq.in.

RECUPERATOR LAYOUT.

(1) Ratio of compression m :

The ratio of compression is based primarily on a consideration of proper counter recoil functioning. A high ratio of compression decreases the size of the recuperator and accelerates the counter recoil at max. elevation, while on the other hand a large value of m causes undue heating which is injurious to the packing in

the floating piston, and further limits the horizontal stability.

m should be taken at from, 1.5 to 1.7

(2) Ratio of recuperator cylinders:

If A_v = effective area of recuperator piston (sq.in)

A_a = cross section of air cylinder (sq.in)

$m = \frac{P_{af}}{P_{ai}}$ = ratio of compression = 1.5 to 1.7

$r = \frac{A_a}{A_v}$ = ratio of recuperator cylinder

l = length of air column in terms of cross section area of air cylinder.

$j = \frac{l}{b}$ = length of air column in terms of recoil stroke

Then, the initial volume,

$$V_o = A_v\, b\, \frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1} \quad \text{where } k = 1.3$$

and since $\frac{A_a}{A_v} = r = \frac{l}{b} = J$

$$rj = \frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1}$$

Let j = 1 approx. hence $r = \frac{m^{\frac{1}{k}}}{m^{\frac{1}{k}} - 1} = \frac{A_a}{A_v}$

To compute A_v we have $F_{vi} = 1.3W_r(\sin\phi_m + u\cos\phi_m)$ (lbs)

hence $A_v = \frac{1.3\ W_r}{P_{ai}}(\sin\phi_m + u\cos\phi_m)$ sq.in.

where P_{ai} = 1500 lbs/sq.in. (approx.)

Then $A_a = A_v\ r$ sq.in. A further limitation for r, is that

$r = \frac{V_{max}}{12}$ where V_{max} = max. vel. of recoil

12 ft/sec. = max. allowable vel. of floating piston

LAYOUT OF CALCULATIONS FOR COUNTER RECOIL.

(1) During the constant orifice period,

$$A_v\left(p_a - \frac{R_f}{A_a}\right) - \frac{c^2 A_v^3 v^2}{175\, w_o^2} - \Sigma R = m_r\, v\, \frac{dv}{dx}$$

where A_v = effective area of the recuperator piston (sq.in)

p_a = air pressure (lbs/sq.in)

R_f = floating piston friction (lbs)

A_a = area of cross section of air cylinder (sq.in)

C = reciprocal of contraction factor of constant orifice = $\frac{1}{0.7}$

w_o = area of constant orifice in recuperator system (sq.in)

ΣR = total friction - guide + packing (lbs)

The max. velocity counter recoil is attained shortly after the beginning of counter recoil. If

p_{af} = final air pressure (lbs/sq.in)

p_{ai} = initial air pressure (lbs/sq.in)

Then $p_a = p_{ai} + 0.9(p_{af} - p_{ai})$

$$= \frac{p_{ai}}{10}(9m+1) = 0.95\, m\, p_{ai} \text{ approx.}$$

where $m = \frac{p_{af}}{p_{ai}} = 1.5$ to 1.7

At max. vel. $m_r v \frac{dv}{dx} = 0$ · Assuming a max.velocity at horizontal recoil = 3 ft/sec.

$$\frac{C^2 A_v^3 v^2}{175 w_o^2} = A_v\left(p_a - \frac{R_f}{A_a}\right) - \Sigma R \text{ and } W_o^2 = \frac{C^2 A_v^3 v^2}{175\left[A_v\left(p_a - \frac{R_f}{A_a}\right) - \Sigma R\right]}$$

$$\text{hence } w_o = \frac{C\, A_v^{\frac{3}{2}}}{13.2\sqrt{A_v(p_a - \frac{R_f}{A_a}) - \Sigma R}} \quad \text{sq.in.}$$

The constant orifice should be designed for counter recoil at horizontal or minimum elevation.

As a numerical illustration, the value of the constant orifice in one model has been worked out as follows:

$$(P_a - \frac{R_f}{A_a})A_v - \frac{KV^2}{W_o^2} = \Sigma R$$

$$(p_a - \frac{.352P_a+3430}{22.88})6.24 - \frac{K^2\ A^3\ V^2}{175W_o^2} = .253\ p_a + 1810$$

$$6.24P_a - .0961P_a - 936.68 - \frac{15.4267}{W_o^2} = .253P_a + 1810$$

$$5.891P_a - 2746.68 = \frac{15.4267}{w_o^2} \quad (P_a = 2080 \text{ lbs})$$

$$12253.28 - 2746.68 = W_o^2 = \frac{15.4267}{9506.6} \quad .0016$$

w_o = .04 sq.in. from drawings = .03896

Percentage accuracy 2.57

Compute velocity of the recoiling parts when buffer is entering V_o = 2.35 ft/sec.

(2) Retardation period of counter recoil.

In the retardation period, we will assume a constant unbalanced force acting in the retardation. Let

ϕ = total unbalanced force in counter recoil retardation (lbs)

v_o = entrance velocity into buffer chamber (ft/sec)

k = factor of safety 0.95

$$B' = \frac{C' A_b^3 v^2}{175\ w_{xb}^2}$$

l_b = length of buffer (ft)

x_b = displacement in buffer (ft)

p_a' = pressure on oil side of floating piston in

air cylinder (lbs/sq.in)

p_v = pressure in recuperator (lbs/sq.in)

A_v = effective area of recuperator piston (sq. in)

then $\phi = B' + \Sigma R - p_v A_v = B' + \Sigma R - (p'_a - \frac{C'^2_o A^2_v v^2}{175 w^2_o}) A_v$

$$= B' + \Sigma R - p'_a A_v + \frac{C'^2_o A^3_v v^2}{175 w^2_2}$$

$$= (\frac{C'^2 A^3_b}{W^2_{xb}} + \frac{C'^2_o A^3_v}{w^2_o}) \frac{v^2}{175} + \Sigma R - p'_a A_v$$

Further $\phi = \frac{\frac{1}{2} m_r v^2_o}{k l_b}$ (lbs) and $\phi_{xb} = \frac{m_r}{2}(v^2_o - v^2_{xb})$

hence $v^2_{xb} = v^2_o - \frac{2\phi x_b}{m_r} = \frac{v^2_o}{k l_b}(k l_b - x_b)$ Hence the equation of counter recoil during the retardation period, becomes,

$$= \frac{\frac{1}{2} m_r v^2_o}{k l_b} = (\frac{c'^2 A^3_b}{w^2_{xb}} + \frac{c'^2_o A^3_v}{w^2_o})(\frac{k l_b - X_b}{175}) \frac{v^2_o}{k l_b} + \Sigma R - p'_a A_v$$

$$\text{or } [(\frac{c'^2 A^3_b}{w^2_{xb}} + \frac{c'^2_o A^3_v}{w^2_o})(k l_b - X_b) - \frac{m_r}{2}] \frac{v^2_o}{k l_b} = p'_a A_v - R$$

The only variable being w_{xb} which therefore, becomes a function of xb as we should expect.

To solve for w_{xb} we have the following expression in terms of x_b, the displacement in the counter recoil in the buffer:

$$W^2_{xb} = \frac{C'^2 A^3_b (k l_b - x_b) v^2_o}{175 k l_b [p'A - R - (\frac{C'^2 A^3_v}{w^2_o} \frac{(k l_b - x_b)}{175} - \frac{m_r}{2}) \frac{v^2_o}{k l_b}]}$$

$$\text{hence } W_{xb} = \frac{C' A^{\frac{3}{2}}_b v_o}{13.2} \sqrt{\frac{(k l_b - x_b)}{k l_b [p' A_v - R - (\frac{C'^2 A^3_v}{w^2_o} \frac{(k l_b - x_b)}{175}}}$$

(sq.in)

$$- \frac{m_r}{2}) \frac{v_o^2}{k\ l_b}]$$

Obviously $p_a' = p_a - \frac{R_f}{A_a}$, as a function of x_b and its proper value must be used for a given x_b.

Computation of v_o:

Knowing w_o from (1), we may readily calculate v_o by the logarithmic method, as follows:

$$\log(A - \frac{C_o'^2 A_v^3 v_2^2}{175 w_o^2}) = \log(A - \frac{C_o'^2 A_v^3 v_1^2}{175 w_o^2}) - \frac{C'^2 A^3 (x_2 - x_1)}{200\ m_r w_o^2}$$

where $A = p_a' A_v - \Sigma R$

Only few intervals are needed, the mean air pressure being taken for these intervals.

The following is an illustration of actual calculations made for the entrance into the buffer on counter recoil.

LAYOUT CALCULATIONS OF BUFFER.

V_o = velocity of counter recoil when buffer enters

$R\ b\ c\ \phi = \frac{1}{2} m_a V_o^2$ = energy to be dissipated during buffer action

where b = length of buffer in inches = 12 in.

R = a factor of safety .95

$$c\phi = \frac{\frac{1}{2} m\ V_o^2}{Rb} = \frac{\frac{1}{2} 284 \times 2.38^2}{k \times 12} = 73.5$$

$$\phi = - MV \frac{dv}{dx}$$

$$x_b = \frac{m}{2}(V_o^2 - V^2) = \frac{\quad}{2.35}^2 - \frac{2 \times 73.5}{284} x$$

$V^2 = 5.57 - .517\ x_b$

C'recoil	x_b	v	v	
60	1.22	2.22	2.18	1.8
62	2.	1.97	1.93	1.8
64	2.	1.69	1.69	0
66	2.	1.36	1.31	3.7
68	2.	.89	.93	5.
70.78	2.88	0	0	0

FORCES (Buffer forces)

$$\phi = \Sigma R + \frac{K' A_b^3 V^2}{175 W_x^2} - [(P - \frac{R_f}{A_a})A_v - \frac{K A_v^3 V^2}{175 W_o^2}]$$

$$= (.253P_a + 1810) + \frac{K' A_b^3 V^2}{175 W_x^2} - (P_a A_v - \frac{.352P_a + 3430}{22.85} A - \frac{K A_v^3 V^2}{175 W_o^2})$$

$$= 0.253P_a + 1810 + \frac{\overline{1.5}^2 \times \overline{4.38}^3 V^2}{175\ W_x^2} - (6.24P_a - 0.0963P_a - 938 - \frac{K\ A_v^3 V^2}{175\ W_o^2})$$

$$= 0.253P_a + 1810 + 1.0804 \frac{V^2}{W_x^2} - (6.3363P_a - 938 - \frac{\overline{1.1}^2 \times \overline{6.24}^3 \times V^2}{175 \times \overline{.0389}^2})$$

$$= 0.253P_a + 1810 + 1.0804 \frac{V^2}{W_x^2} - 6.3383P_a + 938 + 1132.74\ V^2$$

$$68.78 = -6.0833P_a + 2740 + \frac{1.0804}{W_x^2} V^2 + 1132.74\ V^2$$

$$W_x^2 = \frac{1.0804\ V^2}{6.08P_a - 2570 - 1132.74V^2}$$

C'R	V	V^2	$1.08V^2$	P_a	$6.08P_a$	1132	$747V_a^2$	W_x^2	W_x	Actual areas.
60	2.22	4.53	4.90	1520	9250	5550	1030	.00475	.0438	.0395
62	1.98	3.92	4.24	1500	9130	4440	2020	.00210	.0438	.0395
64	1.70	2.89	3.13	1480	9000	3270	3000	.00102	.0319	.0305
66	1.35	1.83	1.97	1470	8940	2060	4210	.00047	.0217	.0213
68	.95	.90	.97	1450	8820	1020	5150	.00019	.0138	.0130

FILLING IN OIL HOLES FROM RECOIL CYLINDER INTO BUFFER CHAMBER.

Area of the filling in oil holes must be large enough to fill the hydraulic cylinder during recoil period.

P = average pressure = average hydraulic pull = 720 lbs/sq.in.

d = weight of cubic ft. of hydraulic oil = 53 lbs/cu.ft.

V_{max} = maximum velocity of recoil = 35 ft/sec.

b = length of recoil = 5.9 ft.

C = contraction factor = .85

A_1 = effective area of recoil cylinder 26 sq. in. = 26 sq.in.

A_b' = area of the larger buffer chamber (d=90 mm)= .0685 sq.ft.

$$P = \frac{V^2 b}{2g} \qquad t \times P \times A_1 \times M_r\, V_{max}$$

$$t = \frac{M_r V_{max}}{P.A_1}$$

$$C\,W\,V\,t = A_b' t \qquad C\,W\sqrt{\frac{2gp}{b} \times \frac{M_r V}{pA}} = A_b b$$

$$C\,W\sqrt{\frac{2\times 32.2\times 720\times 144}{53}} \times \frac{284\times 35}{720\times 26} = .0685 \times 59.$$

$$.85\times W\times 355\times 631 = .4042$$

$$\frac{2}{(548)^2} + \frac{1}{(.487)^2} = \frac{1}{W^2}$$

$$\frac{2}{.3} + \frac{1}{.9483} = \frac{1}{W^2}$$

$$\frac{1}{W^2} = \frac{2.1966}{.2845}$$

$W^2 = .1295$ $\qquad$ $W = .36$

$.85 \times W \times 355 \times .531 = .4042$

$$W = \frac{4042}{355 \times .531 \times 85} = .002523 \text{ sq.ft.}$$

$W = .002523 \times 144 = .3634$

W from design = .3634

W from draw. = .36 $\qquad$ div. 1%

UNIVERSITY OF CALIFORNIA LIBRARY
Los Angeles
This book is DUE on the last date stamped below.

MAY 7 1954

MAR 3 1958
MAR 10 RECD

Form L9–25*m*-8,'46(9852)444

THE LIBRARY
UNIVERSITY OF CALIFORNIA
LOS ANGELES

UC SOUTHERN REGIONAL LIBRARY FACILITY
A 000 768 739 5

UF
643
U58t

Engineering
Library

AUXILIARY
STACK

SEP '73